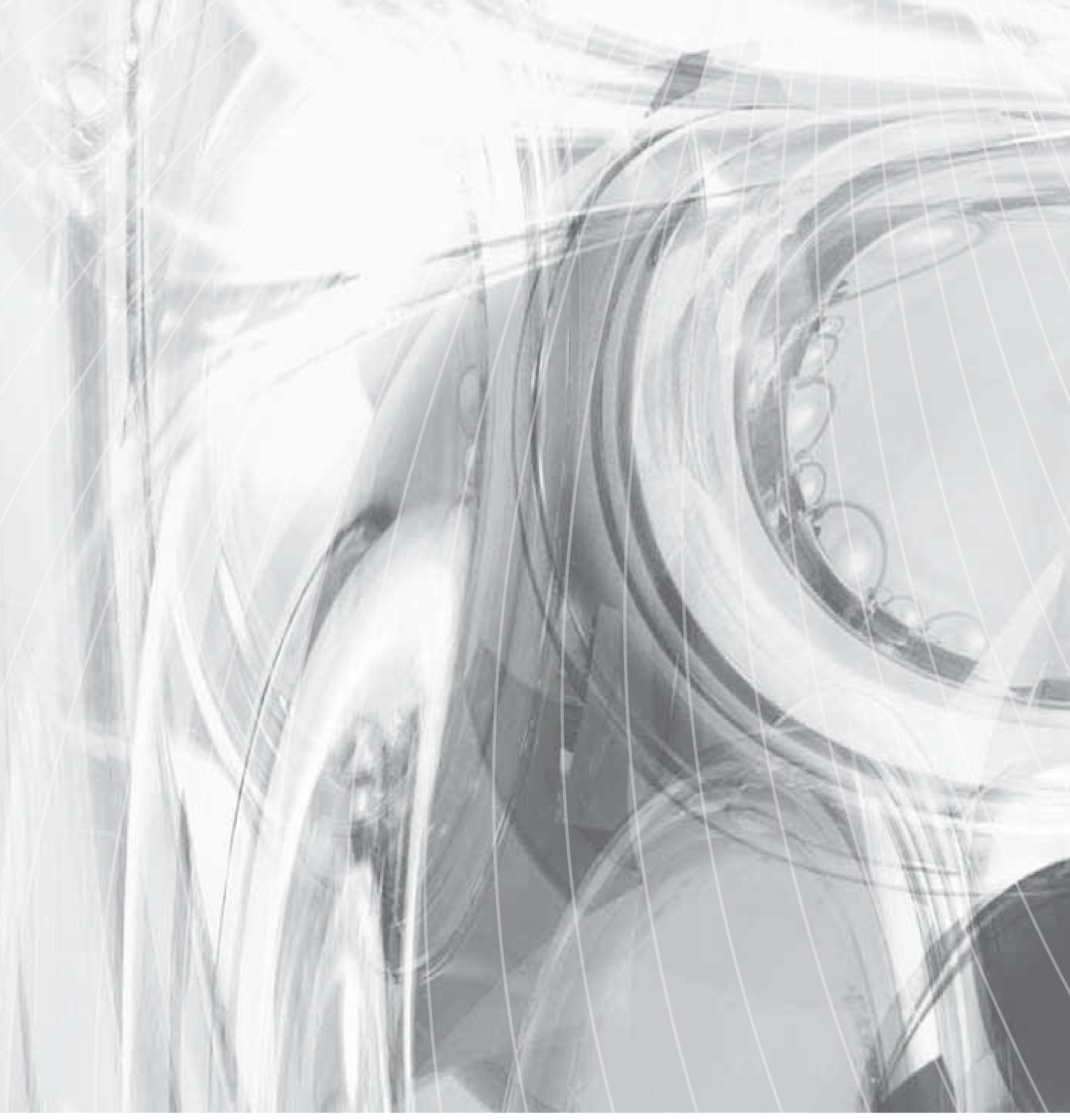
Solución a los ejercicios de geometría y trigonometría



CAPÍTULO 2

EJERCICIO 1

1. 40.1708º
2. 61.7058º
3. 1.03416º
4. 73.6777º
5. 9.1525º
6. 98.3791º
7. 40º 19’ 12”
8. 61º 14’ 24”
9. 18º 15’ 18”
10. 29º 24’ 39”
11. 19º 59’ 24”
12. 44º 00’ 36”

EJERCICIO 2

1. $\frac{7}{6}\pi\ rad = 3.665\ rads$
2. $\frac{5}{3}\pi\ rad = 5.236\ rads$
3. $\frac{5}{4}\pi\ rad = 3.927\ rads$
4. $\frac{5}{2}\pi\ rad = 7.854\ rads$
5. $\frac{2}{5}\pi\ rad = 1.256\ rads$
6. $\frac{5}{9}\pi$ rad $= 1.745\ rads$
7. $\frac{1}{6}\pi\ rad = 0.523\ rad$
8. $\frac{11}{6}\pi\ rad = 5.759\ rads$
9. $\frac{2}{3}\pi\ rad = 2.094\ rads$
10. $\frac{3}{4}\pi\ rad = 2.356\ rads$
11. $\frac{4523}{18000}\pi\ rad = 0.789\ rad$
12. $\frac{1283}{1800}\pi\ rad = 2.239\ rads$
13. $\frac{2711}{3240}\pi\ rad = 2.628\ rads$
14. $\frac{33601}{14400}\pi\ rad = 7.330\ rads$

EJERCICIO 3

1. 120º
2. 330º
3. 135º
4. 240º
5. 1260º
6. 20º
7. 468º
8. 15º
9. 90º
10. 270º
11. 9º 38’ 34”
12. 64º 10’ 37”
13. 360º
14. 28º 38’ 52”

EJERCICIO 4

1. 55º 46’ 50”
2. 40º 13’ 15”
3. 49º 19’ 33”
4. 59º 19’ 45”
5. 108º 7’ 48”
6. 75º 44’ 22”
7. 246º 34’ 15”
8. 875º 11’40”
9. 383º 51’ 21”
10. 227º 3’ 18”
11. 4º 33’ 11”
12. 15º 41’ 18”
13. 3º 21’ 41”
14. 13º 15’ 18”

EJERCICIO 5

1. Suplementarios
2. Complementarios
3. Conjugados
4. Conjugados
5. Conjugados
6. Complementarios
7. Suplementarios
8. Complementarios
9. Conjugados
10. Suplementarios
11. 10º
12. 57º
13. 80º
14. 30º
15. 18º
16. 20º
17. 36º
18. 120º
19. *a)* $\angle COB = 30º$, $\angle BOA = 60º$
    b) $\angle AOB = 45º$, $\angle BOC = 30º$, $\angle COD = 15º$
    c) $\angle AOB = 50º$, $\angle DOB = 130º$
    d) $\angle AOB = 65º$, $\angle BOC = 45º$, $\angle COD = 70º$
    e) $\angle AOB = 30º$, $\angle BOC = 90º$, $\angle COD = 60º$
    f) $\angle AOB = \angle COD = 45º$, $\angle BOC = 55º$, $\angle DOE = 35º$
    g) $\angle AOB$ (convexo) = 134º, $\angle AOB$ (cóncavo) = 226º
    h) $\angle AOB$ (convexo) = 50º, $\angle AOB$ (cóncavo) = 310º

EJERCICIO 6

1. 135º
2. 115º.
3. $\theta = 25º$, $\alpha = 30º$
4. O63º 18’S, S26º 42’O
5. 22º 30’
6. 115°
7. 292° 30’
8. 12:30 *h*
9. 48 π *rad*
10. 3:40 *h*

CAPÍTULO 3

EJERCICIO 7

1. $x = 60º$, $\angle a = 60º$, $\angle b = 120º$
2. $x = 46.5º$, $\angle a = \angle b = \angle e = 46.5º$, $\angle c = \angle d = \angle f = 133.5º$
3. $x = 40º$, $\angle a = \angle b = \angle e = 80º$, $\angle c = \angle d = \angle f = 100º$
4. $\angle a = \angle c = 137º$, $\angle b = 43º$
5. $\angle a = \angle c = \angle d = \angle g = 47º$, $\angle b = \angle e = \angle f = 133º$
6. $x = 25º$
7. $x = 26º$, $\angle a = 128º$, $\angle b = 52º$
8. $\angle 10 = \angle 4 = \angle 7 = 70º$, $\angle 1 = \angle 13 = \angle 16 = 110º$
9. $x = 115º$, $y = 65º$
10. $x = 40º$, $y = 110º$
11. $x = 80º$, $y = 60º$
12. $R = 120º$
13. $\angle a = \angle c = \angle e = \angle f = 126º$, $\angle b = \angle d = 54º$
14. $\angle n = \angle z = 50º$, $\angle m = \angle s = \angle y = \angle r = 130º$
15. $\angle x = \angle q = \angle p = \angle k = 35º$, $\angle y = \angle r = \angle s = 145º$
16. $\angle q = \angle z = \angle y = 60º$, $\angle r = \angle w = \angle p = 120º$
17. *a*), *b*), *d*) y *f*)

CAPÍTULO 4

EJERCICIO 8

1. 105°, 110°
2. 10°, 80°
3. 80°, 80°, 20°
4. 55°, 41°
5. 118º, 38º y 24º; 68º, 70º y 42º
6. $\theta = 54º$ y $\beta = 72º$
7. $\angle A = 35º$, $\angle B = 95º$, $\angle C = 50º$
8. $ABC = 69º$, $BCA = 73º$, $BAC = 38º$, $ACD = 107º$, $CDA = 35º$, $CAD = 38º$

EJERCICIO 9

1. Teorema II (*LAL*) $x = 85°$ $y = 12$
2. Teorema III (*ALA*) $x = 13$ $y = 19.8$
3. Teorema I (*LLL*) $x = 32°$ $y = 62°$

EJERCICIO 10

1 a 8. No se incluye la solución por ser demostraciones.

EJERCICIO 11

1. $a = 36º, b = 8º$
2. $x = 15, y = 45$
3. $x = 15º, y = 20º$
4. $x = 25, y = 14$
5. $a = 12º, b = 25º$

EJERCICIO 12

1. $x = 3$
2. $x = 7.2$
3. $x = \pm 9$
4. $x = 7, x = 0$
5. $x = \pm\ 4\sqrt{2}$
6. $x = 2$
7. $x = \pm 6y$
8. $x = \pm 5$
9. $x = \pm 4$
10. $x = 3$

EJERCICIO 13

1. $a' = 3, c' = 5$
2. $a = 30, b' = 16$
3. Lados 12 y 22; $x = 11, y = 36$
4. Lados 8 y 4; $x = 7, y = 5$
5. Lados 8 y 6; $u = 3, t = 10$
6. Lados 10 y 9; $x = 5, y = 3$

EJERCICIO 14

1. $x = 10$
2. $x = \frac{9}{2}$
3. $x = 6$
4. $x = \frac{12}{5}$
5. $x = \frac{25}{3}$
6. $x = 16$
7. $x = 4$
8. $x = \frac{27}{22}$
9. $x = 10$
10. $x = 30$

EJERCICIO 15

1. 68 *m*
2. 481.6 *m*
3. 160 *m*
4. 15 *m*
5. *a*) 28 *m*
   b) 120 *m*

EJERCICIO 16

1. $c = 25$
2. $c = \sqrt{41}$
3. $c = 4\sqrt{5}$
4. $c = 7\sqrt{2}$
5. $b = 16$
6. $a = 2\sqrt{7}$
7. $c = 8$
8. $b = 5\sqrt{2}$
9. $c = 3\sqrt{5}\ m$
10. $b = 5\ m$
11. $c = \sqrt{421}\ cm$
12. $a = 5\sqrt{7}\ dm$
13. Obtusángulo
14. Rectángulo
15. Acutángulo
16. Rectángulo
17. Rectángulo
18. Obtusángulo
19. Rectángulo
20. Acutángulo
21. Rectángulo
22. *a*) $2\sqrt{15}$, *b*) $5\sqrt{13}$, *c*) $2\sqrt{10}$, *d*) $6\sqrt{21}$ *e*) $\frac{40}{3}$, *f*) $\frac{91\sqrt{218}}{218}$, *g*) $\frac{169}{60}\sqrt{30}$

EJERCICIO 17

1. $100\sqrt{73}\ m$
2. $2\sqrt{5}\ m$
3. $40\ cm$
4. $5\sqrt{3}\ cm$
5. $4\sqrt{2}\ m$
6. $\sqrt{91}\ m$
7. $5\ cm$
8. $8\sqrt{3}\ cm$
9. $9\sqrt{2}\ km$
10. $5\sqrt{2}\ cm$
11. $\frac{2\sqrt{2}}{3}m,\ \frac{m}{3}$
12. $2\sqrt{\frac{4m^2 - n^2}{15}},\ 2\sqrt{\frac{4n^2 - m^2}{15}}$ y $2\sqrt{\frac{m^2 + n^2}{5}}$

CAPÍTULO 5

EJERCICIO 18

1. $\angle A = \angle C = 140°, \angle B = 40°$
2. $\angle DCA = 40°, \angle CAD = 60°, \angle DAB = \angle DCB = 100°, \angle D = \angle B = 80°$
3. $\angle ADC = \angle B = 110°, \angle A = \angle C = 70°$
4. $x = 30°, z = 120°, y = 60°$
5. $x = 127°, y = 53°$
6. $x = 120°, y = 55°, z = 125°$
7. $x = 60°, y = 120°, z = 60°$
8. $x = 15°, y = 70°, z = 110°$

EJERCICIO 19

1 a 6. No se incluye la solución por ser demostraciones.

EJERCICIO 20

1. $x = 4\ cm$
2. 4 y 8 *u*
3. 41 *u*
4. $\angle NPO = 24º$
5. $x = 20º$, $y = 68º$
6. $\overline{AB} = 11\ cm$
7. $\overline{MN} = 20\ u$
8. $\overline{AB} = a,\ \overline{IJ} = b$
9. $\overline{AE} = 5$

CAPÍTULO 6

EJERCICIO 21

1. $d = 8$
2. Icoságono
3. $d = 7$
4. Dodecágono
5. Nonágono
6. *a*) 170, *b*) 54, *c*) 27, *d*) 9, *e*) 90, *f*)14, *g*) 104, *h*) 135, *i*) 44
7. Heptágono
8. Hexadecágono
9. Heptadecágono
10. Nonadecágono
11. Heptágono
12. Undecágono
13. Pentágono
14. Tridecágono
15. Dodecágono
16. Octágono
17. Icoságono

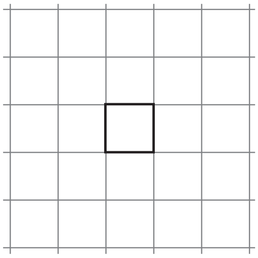
Ejercicio 22

1. *a*) 120°, *b*) 135°, *c*) 150°, *d*) 162°, *e*) 160°, *f*) 171° 25’42”
2. *a*) 540°, *b*) 1 440°, *c*) 2 340°, *d*) 1 080°, *e*) 1 980°, *f*) 6 300°
3. Nonágono (nueve lados)
4. Heptágono (siete lados)
5. Hexadecágono (16 lados)
6. Undecágono (11 lados)
7. Hexágono (seis lados)
8. Hexadecágono (16 lados)
9. Nonágono (nueve lados)
10. Dodecágono (12 lados)
11. Octágono (ocho lados)
12. Triángulo
13. Hexágono (seis lados)
14. Pentadecágono (15 lados)
15. Nonágono (nueve lados)
16. Pentágono (cinco lados)
17. 54°, 129.6°, 129.6°, 108° y 118.8°
18. 110°, 100°, 115°, 135° y 80°
19. 30°, 60°, 90°, 120°, 150°, 210° y 240°
20. $\angle A = 70°$, $\angle B = 65°$, $\angle C = 10°$, $\angle D = 110°$ y $\angle E = 105°$
21. $\angle A = 54°$, $\angle B = 64°$, $\angle C = 116°$, $\angle D = 64°$, $\angle E = 17°$ y $\angle F = 45°$

Capítulo 7

Ejercicio 23

1.



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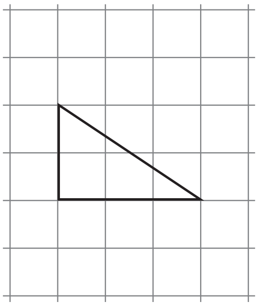

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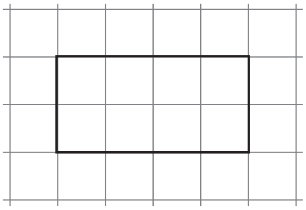

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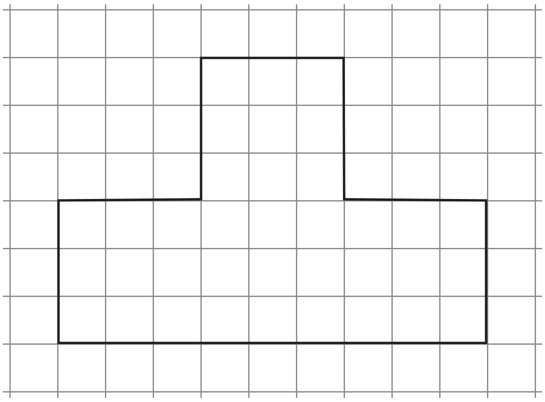

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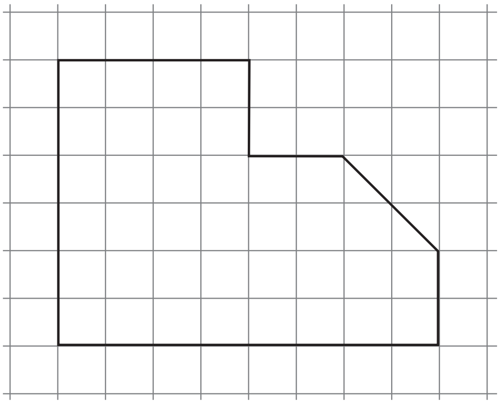

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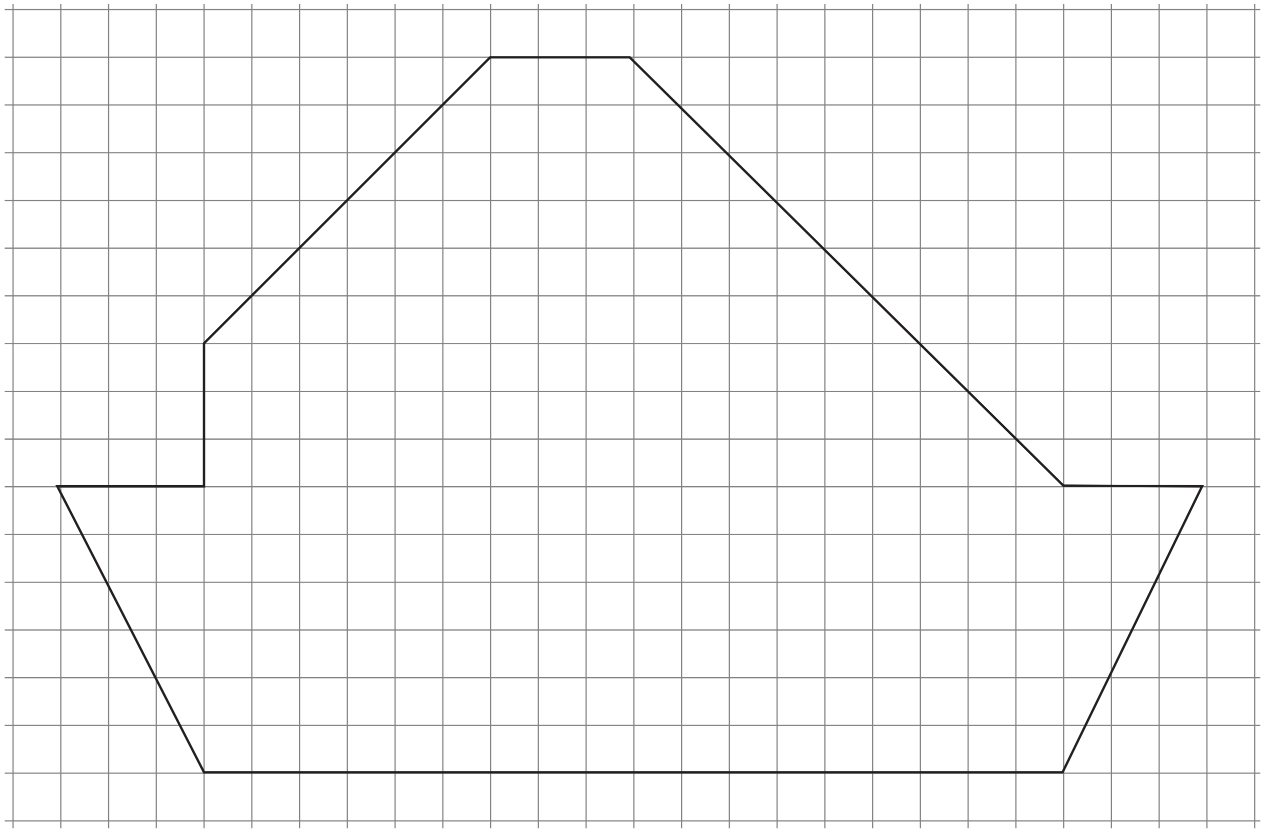

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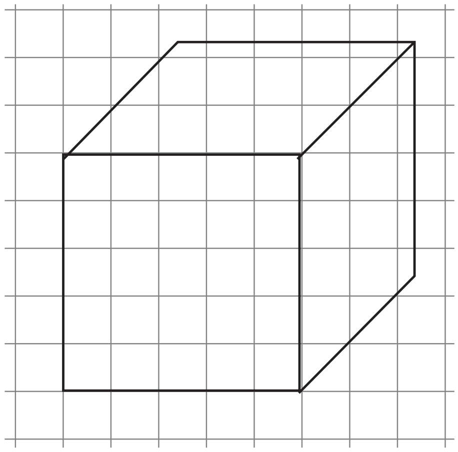

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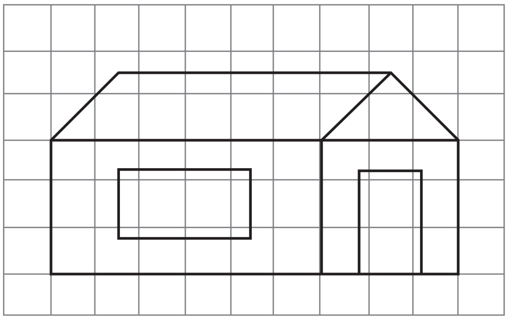

EJERCICIO 24

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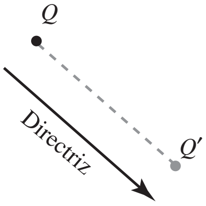

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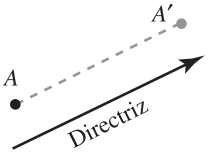

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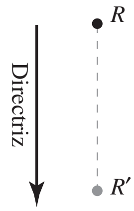

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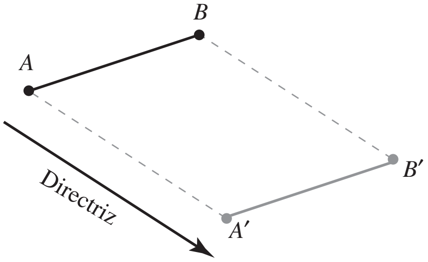

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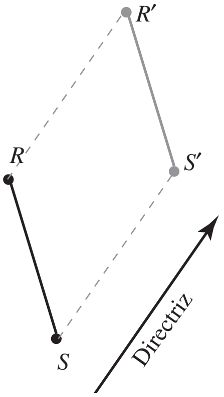

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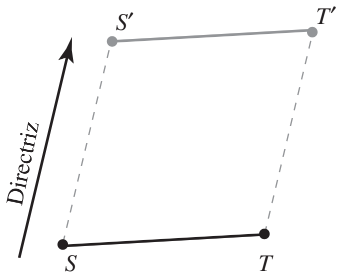

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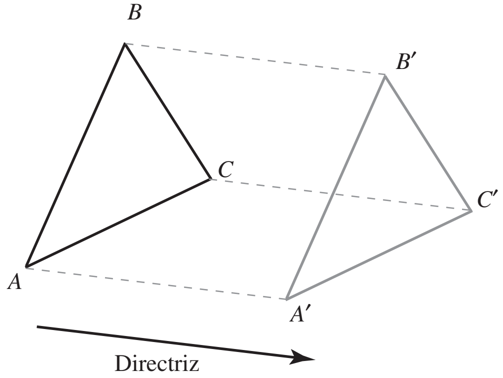

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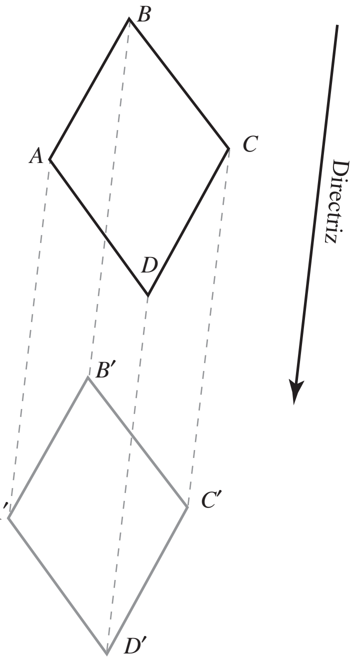

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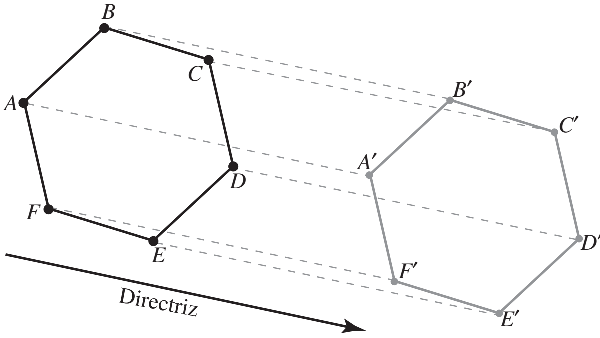

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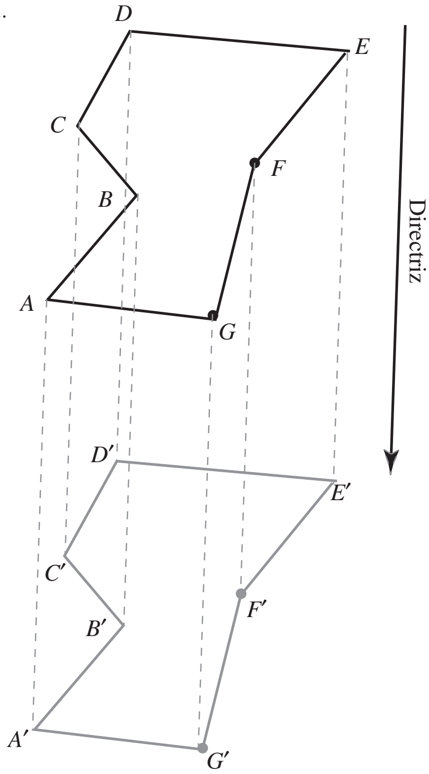

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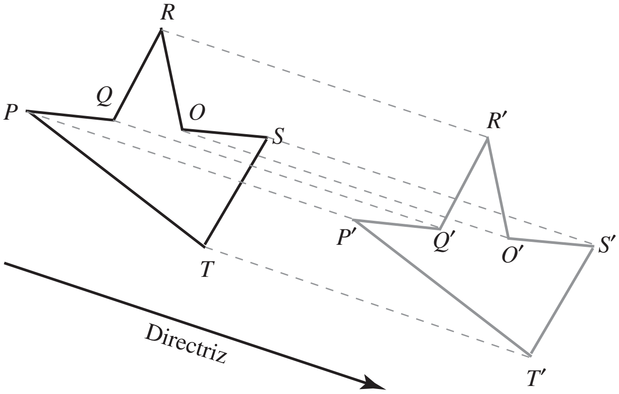

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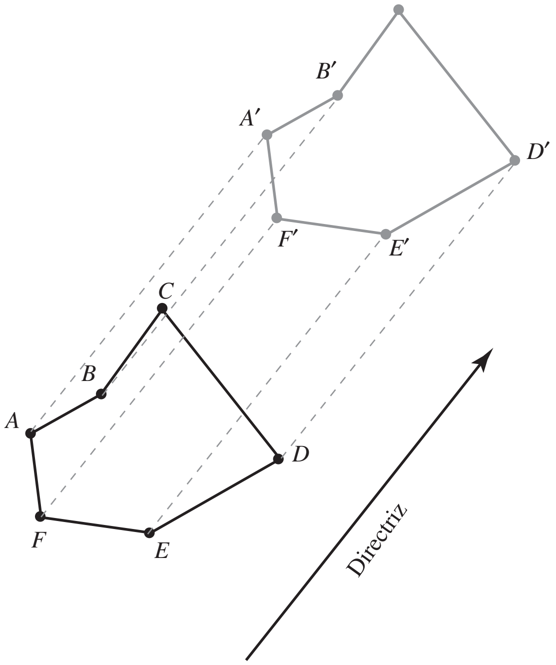

Ejercicio 25

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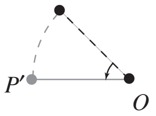

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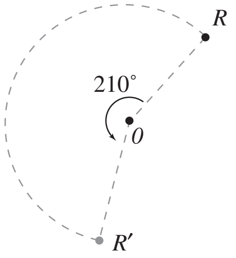

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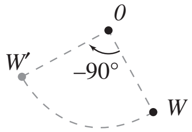

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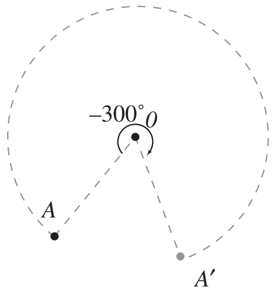
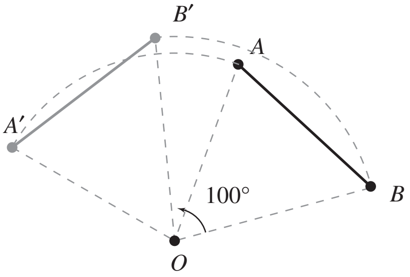
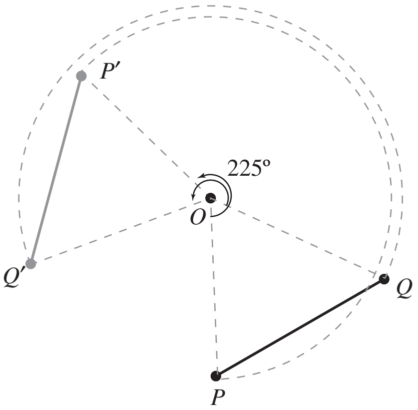

–300°
O
A
A′

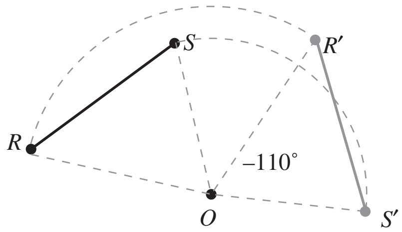
5.


B′
A
A′
100°
B
O

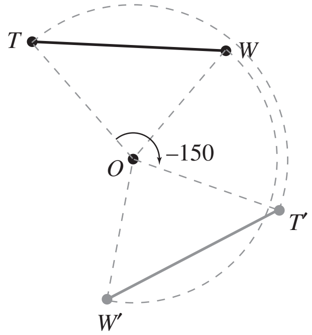
6.


P′
225º
O
Q′
Q
P

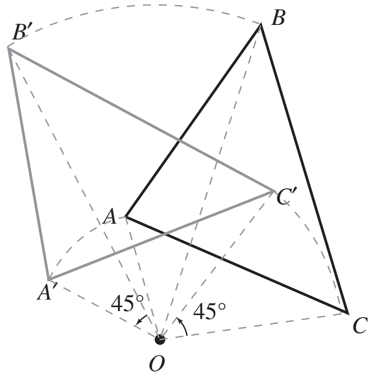
7.


S
R′
R
–110°
O
S′

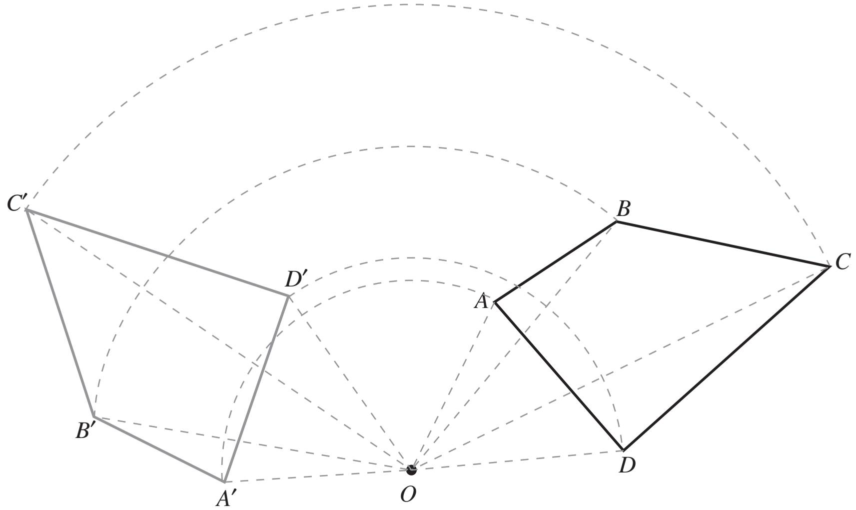
8.


T
W
O
–150
T′
W′

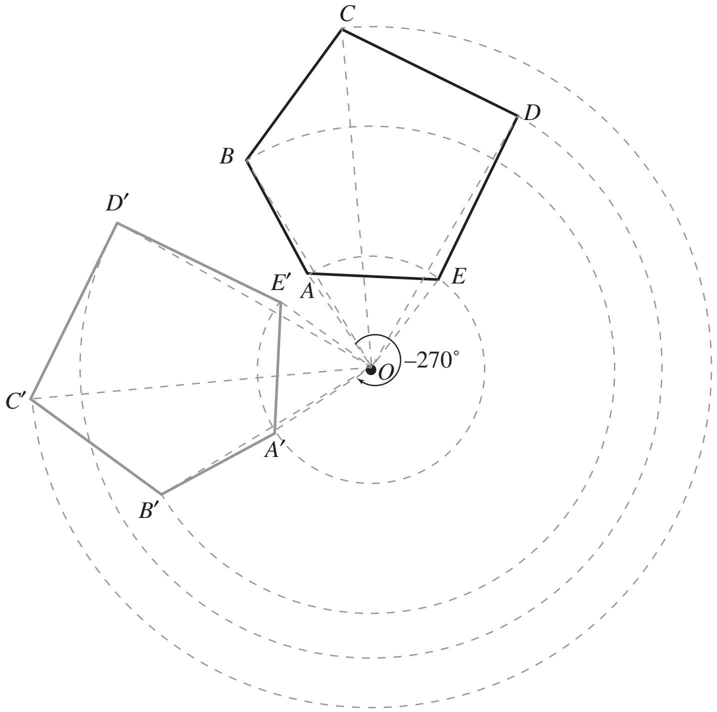
9.


B′
B
A
C′
A′
45°
45°
C
O

10.


C′
B
C
D′
A
B′
A′
O
D

11.



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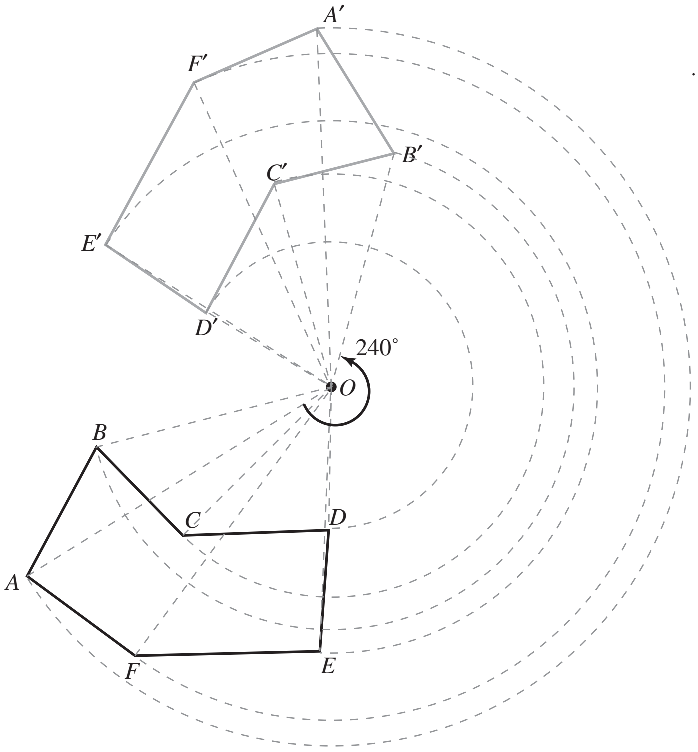

EJERCICIO 26

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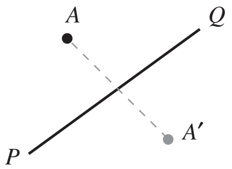

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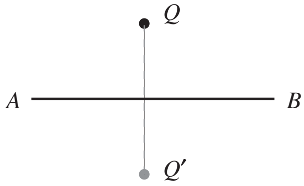

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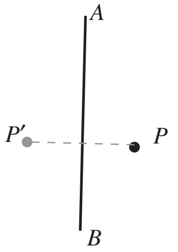

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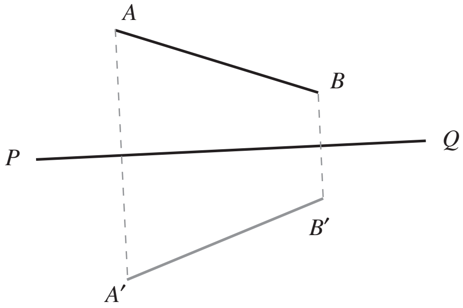

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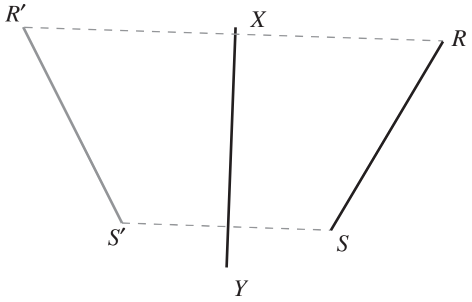

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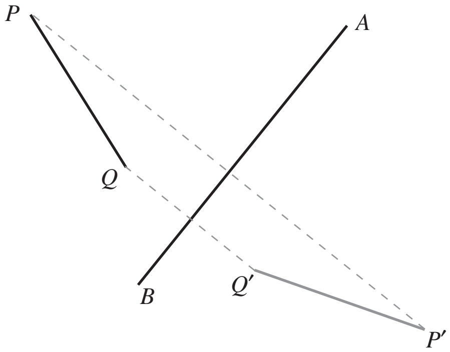

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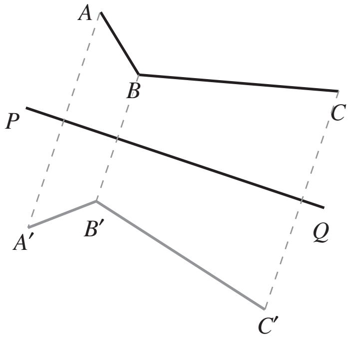
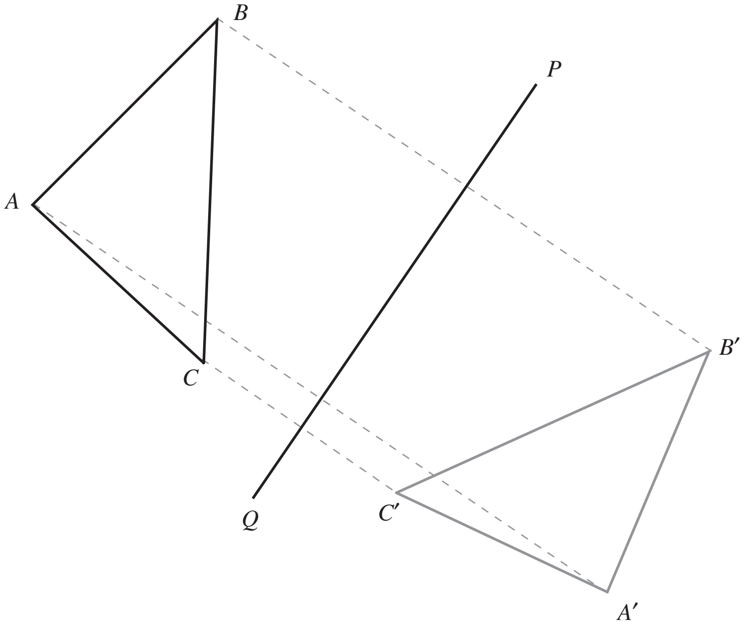
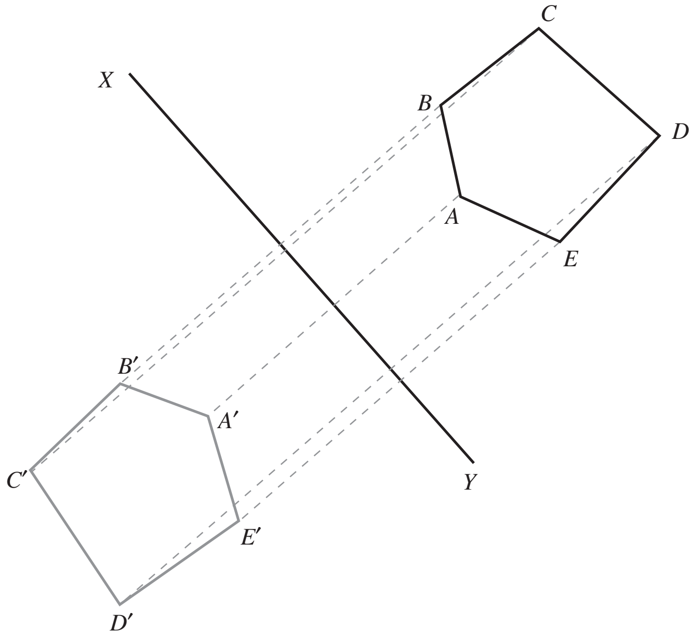


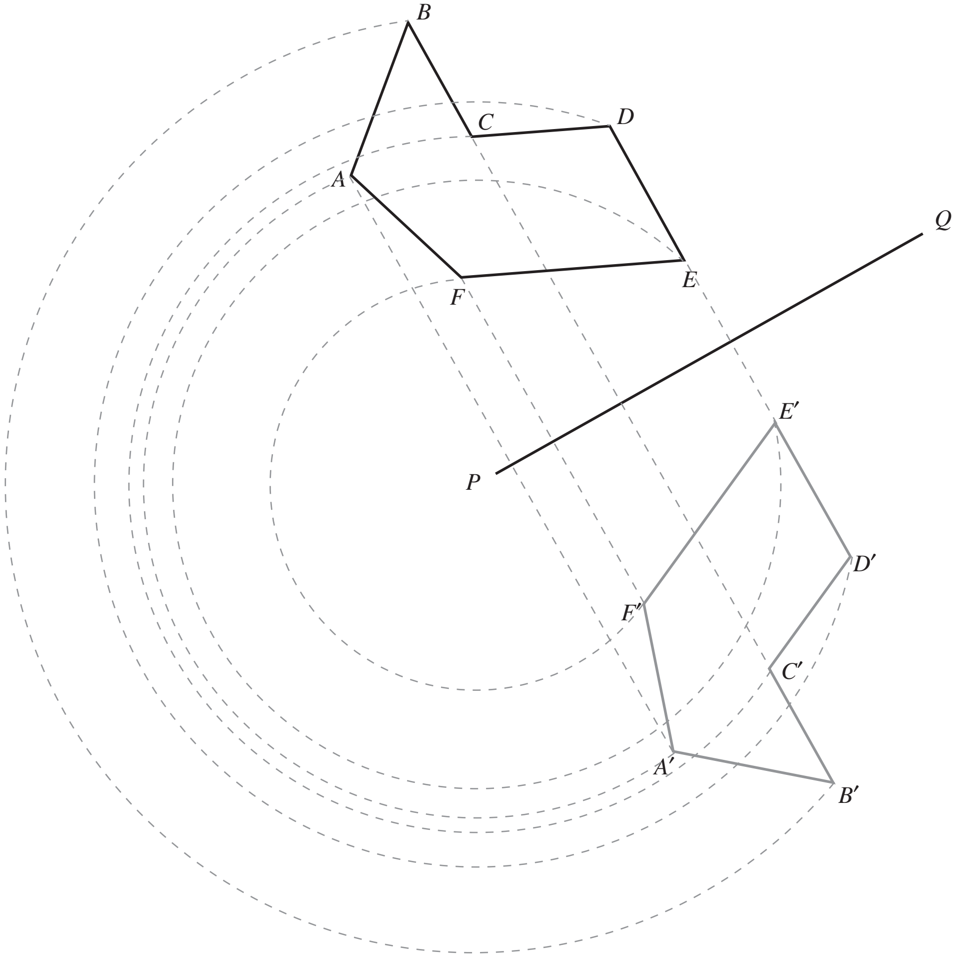
8.



9.

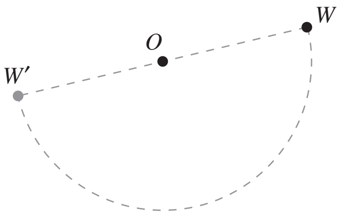


10

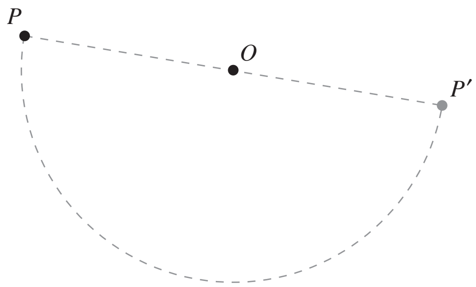


Ejercicio 27

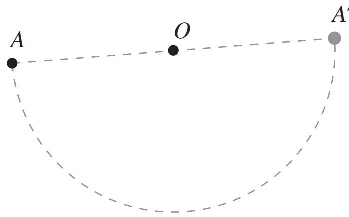
1.



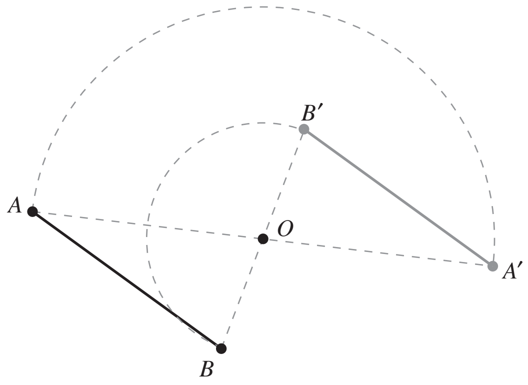
2.



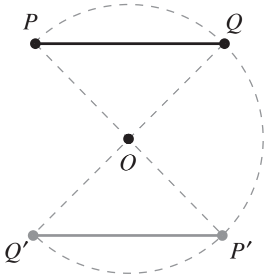
3.



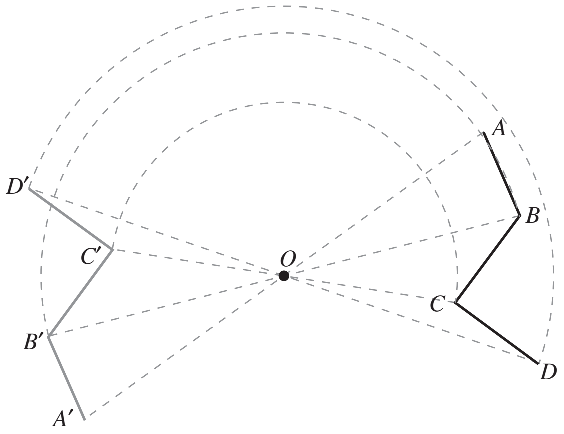
4.



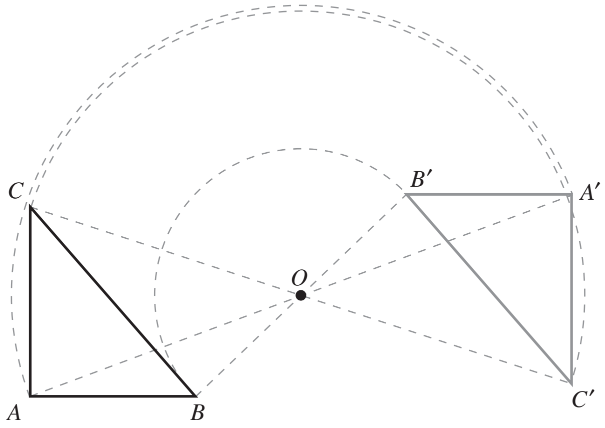
5.



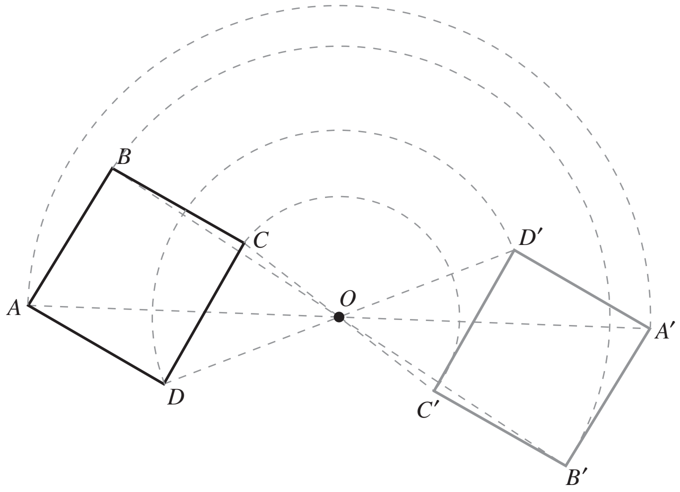
6.



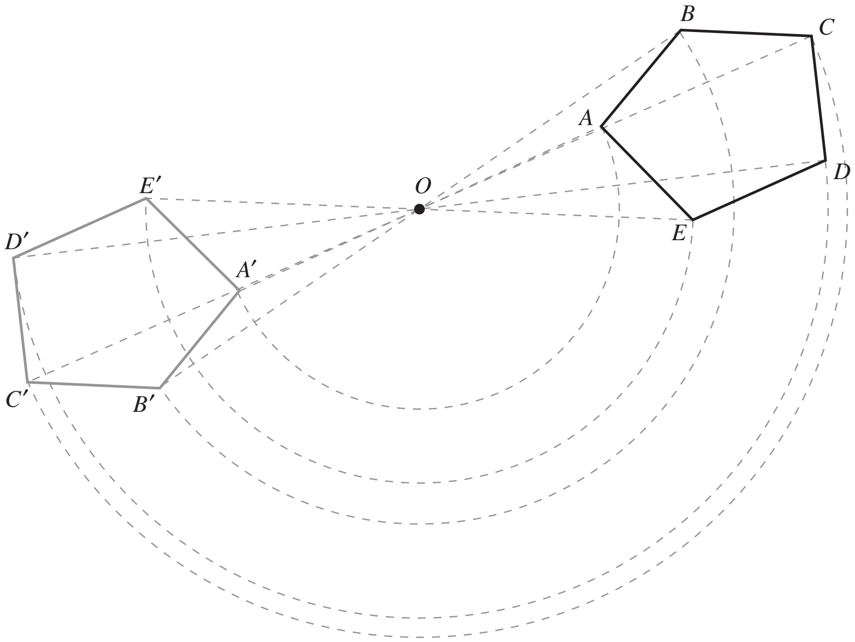
7.



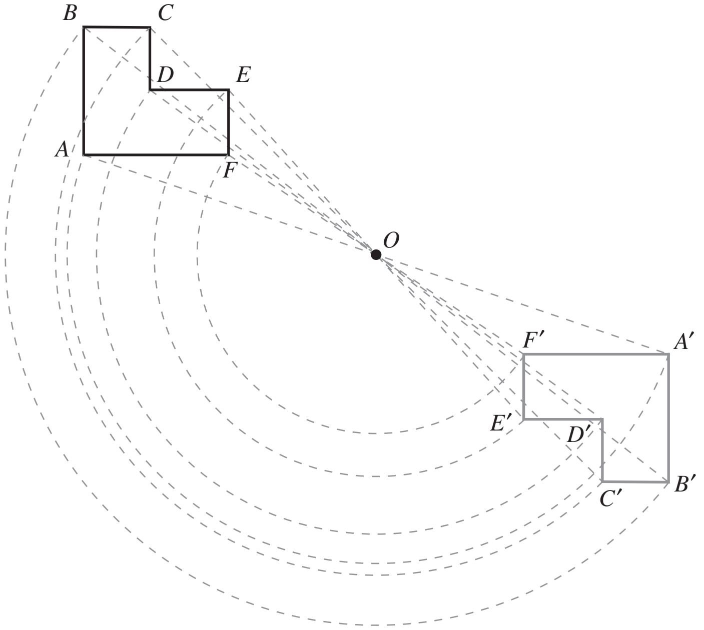
8.



9.



10.



CAPÍTULO 8

EJERCICIO 28

1. $\angle ABC = 30°$, $\angle AOC = 60°$, $\angle BOC = 104°$, $\overset{\frown}{AD} = 116°$
2. $\angle a = 75°$, $\angle b = 50°$, $\angle c = 55°$, $\angle d = 55°$, $\angle e = 50°$, $\angle f = 75°$
3. $\angle ABC = 27.5° = 27°30'$
4. $\angle ABC = 85°$, $\angle DBA = 95°$
5. $\angle A = 105°$, $\angle B = 95°$, $\angle C = 75°$, $\angle D = 85°$
6. *a)* $\angle A = 30°$, *b)* $\angle A = 40°$
7. $\angle a = 60°$, $\angle b = 15°$, $\angle c = 25°$, $\angle d = 30°$, $\angle e = 50°$
8. *a)* $\angle A = 15°$, *b)* $\angle A = 40°$, *c)* $\angle A = 30°$, *d)* $\widehat{a} = 35°$
   e) $\widehat{c} = 120°$, *f)* $\widehat{c} - \widehat{a} = 140°$, *g)* $\widehat{a} = 70°$, *h)* $\widehat{a} = 40°$
9. $\angle u = 120°$, $\angle x = 60°$, $\angle y = 30°$, $\angle w = 60°$, $\angle z = 90°$
10. $\angle a = 90°$, $\angle b = 90°$, $\angle c = 90°$, $\angle d = 90°$, $\angle e = 25°$, $\angle f = 25°$,
    $\angle g = 65°$, $\angle h = 65°$, $\angle i = 40°$

EJERCICIO 29

1. *a)* 10.8, *b)* 7.8, *c)* 9.4
2. *a)* 10.09, *b)* 16.2, *c)* 17.29

EJERCICIO 30

1. Exteriores
2. Tangentes exteriores
3. Interiores
4. Secantes
5. Tangentes interiores
6. Tangentes exteriores
7. $3r$
8. $2u$
9. $2\sqrt{3}\ u$
10. 5 *cm*
11. $\overline{C_1C_3} = \frac{7}{18}R$, $\overline{C_1C_2} = \frac{1}{6}R$
12. r
13. $\frac{\sqrt{5}}{2}R$

CAPÍTULO 9

EJERCICIO 31

1. $P = 8.4\ m$, $A = 4.25\ m^2$
2. $P = 24.9\ m$, $A = 29.4\ m^2$
3. $P = 38.6\ m$, $A = 82.5\ m^2$
4. $P = 52.5\ m$, $A = 118.12\ m^2$
5. $P = 40.0\ m$, $A = 110\ m^2$
6. $P = 65.4\ m$, $A = 37.375\ m^2$
7. $P = 36\ cm$, $A = 81\ cm^2$
8. $P = 10\ m$, $A = 6\ m^2$
9. $A = 150\ m^2$
10. $A = (x^2 - 3x + 2)m^2$
11. $A = 63\ dm^2$
12. $A = 17.5\ dm^2$
13. $A = 900\pi\ cm^2$
14. $A = 81\pi\ cm^2$
15. $A = 400\ cm^2$
16. \$ 2.6/m^2
17. \$ 725.5
18. Altura = 36 *m*, base = 27 *m*
19. Altura = 10 *m*
20. 80 círculos, $1280\pi\ cm^2$
21. *a)* $12\sqrt{14}\ u^2$, *b)* $2\sqrt{255}\ u^2$,
    c) $\frac{15}{4}\sqrt{15}\ u^2$
22. $x = 9$, $A = 98\ m^2$
23. *a)* $2\pi\ cm^2$, *b)* $\frac{1}{6}\pi\ cm^2$,
    c) $\frac{9}{4}\pi\ cm^2$, *d)* $\frac{32}{3}\pi\ cm^2$
24. *a)* $(\pi - 2)cm^2$
    b) $\frac{3}{2}\ (\pi - \frac{3}{2}\sqrt{3}\ )cm^2$
    c) $16(\pi - 2)cm^2$

EJERCICIO 32

1. *a)* $\overline{TS} = 24\ cm$, *b)* $\overline{BC} = 13\ cm$, *c)* $P = 44\ cm$, $A = 14\sqrt{11}\ cm^2$
2. $A = 84\ cm^2$
3. $A = 2r^2(4 - \pi)$
4. $A = 3\pi r^2$
6. $A = 25(2\sqrt{3} - \pi)dm^2$
7. $A_s = 25(4 - \pi)\ cm^2$
8. $A_s = 100\pi\ dm^2$
9. $A_s = 64(4 - \pi)\ mm^2$
10. $A_s = 4(10 + \pi)\ dm^2$
11. $A_s = 196(4 - \pi)\ cm^2$
12. $A_s = 1\,152(\pi - 2)\ mm^2$
    $P = 96\pi\ mm$
13. $A_s = 32(6 - \pi)mm^2$
14. $A_s = 128(\pi - 2)\ mm^2$
15. $A_s = 256(4 - \pi)\ cm^2$
16. *a)* $A = 3\sqrt{3}\ dm^2$
    b) $A = 256\sqrt{3}\ dm^2$
17. $A_s = 36\pi\ cm^2$
18. $A_s = \frac{1}{8}\pi\ cm^2$
19. $A_s = 2\ cm^2$,
    $P = 2(1 + \pi)cm$
20. $A_s = \frac{5}{2}\pi\ cm^2$
    $P = (6 + 4\pi)cm$

CAPÍTULO 10

EJERCICIO 33

1. $A_T = 4\sqrt{3}\ cm^2$, $V_T = \frac{2}{3}\sqrt{2}\ cm^3$
2. $A_T = 3\sqrt{3}\ cm^2$, $V_T = \frac{\sqrt{6}}{4}\ cm^3$
3. $A_T = 72\ cm^2$, $V_T = 24\sqrt{3}\ cm^3$
4. $A_T = 150\ cm^2$, $V_T = 125\ cm^3$
5. $A_T = 72\sqrt{3}\ cm^2$, $V_T = 72\sqrt{2}\ cm^3$

6. $A_T = 6\sqrt{3}\ cm^2$, $V_T = \sqrt{6}\ cm^3$
7. $A_T = \left(60\sqrt{25+10\sqrt{5}}\right) cm^2$, $V_T = (350 + 150\sqrt{5})cm^3$
8. $A_T = \left(12\sqrt{25+10\sqrt{5}}\right) cm^2$, $V_T = (30 + 14\sqrt{5})cm^3$
9. $A_T = 15\sqrt{3}\ cm^2$, $V_T = \left(\frac{15\sqrt{3}+5\sqrt{15}}{4}\right) cm^3$
10. $A_T = 250\sqrt{3}\ dm^2$, $V_T = \left(\frac{1875\sqrt{2}+625\sqrt{10}}{6}\right) dm^3$
11. $A_T = 9\sqrt{3}\ cm^2$
12. $V_T = \frac{27\sqrt{6}}{4}\ cm^3$
13. $h = \frac{4\sqrt{3}}{3}\ cm$
14. $V_T = 2\sqrt{2}\ cm^3$
15. $L = \sqrt[3]{2}\ m$, $A_T = 6\sqrt[3]{4}\ m^2$
16. $h = 6\sqrt{2}\ cm$, $A_T = 72\sqrt{3}\ cm^2$
17. $V_T = \frac{4}{3}\ cm^3$
18. $V_T = 36\ cm^3$
19. $V_T = \frac{15\sqrt{2}+5\sqrt{10}}{6}\ cm^3$
20. $V_T = \frac{\left(3+\sqrt{5}\right)\cdot\sqrt[4]{75}\sqrt{A_t^{\,3}}}{180}$

EJERCICIO 34

1. $A_L = 50\ cm^2$, $A_T = 62\ cm^2$, $V_T = 30\ cm^3$
2. $A_L = 72\ cm^2$, $A_T = (72+8\sqrt{3})cm^2$, $V_T = 24\sqrt{3}cm^3$
3. $A_L = 16\ cm^2$, $A_T = 18\ cm^2$, $V_T = 4\ cm^3$
4. $A_L = 97.5\ cm^2$, $A_T = \frac{195+75\sqrt{3}}{2}\ cm^2$, $V_T = \frac{975}{8}\sqrt{3}\ cm^3$
5. $A_L = \left(16\sqrt{2}+8\right) cm^2$, $A_T = \left(24\sqrt{2}+8\right) cm^2$, $V_T = 16\ cm^3$
6. $A_L = 16\ cm^2$, $A_T = 24\ cm^2$, $V_T = 8\ cm^3$
7. $A_L = 64\sqrt{3}\ cm^2$, $A_T = \left(64\sqrt{3}+24\right) cm^2$, $V_T = 96\ cm^3$
8. $A_L = 400\ cm^2$, $A_T = 400\left(2+\sqrt{2}\right) cm^2$, $V_T = 1000\left(1+\sqrt{2}\right)\ cm^3$
9. $A_L = 1200\ cm^2$, $A_T = 300\left(4+\sqrt{5}\right) cm^2$, $V_T = 3000\sqrt{3}\ cm^3$
10. $A_L = 16\sqrt{3}\ cm^2$
11. $V_T = 27u^3$
12. $A_L = 48\ cm^2$
13. $V_T = \frac{21}{4}\ cm^3$
14. $A_T = \frac{180+25\sqrt{3}}{2}\ cm^2$
15. $V_T = \frac{81}{2}\ cm^3$
16. $A_L = 16\left(1+\sqrt{2}\right) cm^2$
17. $V_T = \frac{\sqrt{6A_t^{\,3}}}{36}$
18. $V_T = \frac{\sqrt{A_L^{\,3}}}{8}$
19. $V_T = \frac{27\sqrt{2}}{4}\ cm^3$
20. $A_L = 3\sqrt[3]{3V_t^{\,2}}$

EJERCICIO 35

1. $A_L = 3\sqrt{55}\ cm^2$, $A_T = \left(9+3\sqrt{55}\right) cm^2$, $V_T = 12\ cm^3$
2. $A_L = \frac{3\sqrt{3}}{4}\ cm^2$, $A_T = \sqrt{3}\ cm^2$, $V_T = \frac{\sqrt{2}}{12}\ cm^3$
3. $A_L = 12\sqrt{7}\ cm^2$, $A_T = \left(12\sqrt{7}+7\sqrt{3}\right) cm^2$,
   $V_T = \frac{35\sqrt{3}}{3}\ cm^3$
4. $A_L = 38.4\ cm^2$, $A_T = 64\ cm^2$, $V_T = 81.92\ cm^3$
5. $A_L = 30\pi\ cm^2$, $A_T = 48\pi\ cm^2$, $V_T = 45\pi\ cm^3$
6. $A_L = 32\pi\ cm^2$, $A_T = 64\pi\ cm^2$, $V_T = 64\pi\ cm^3$
7. $A_L = 7\sqrt{150}\ \pi\ cm^2$, $A_T = \left(7\sqrt{150}+49\right)\pi\ cm^2$,
   $V_T = 147\pi\ cm^3$
8. $A_L = 4\sqrt{17}\ \pi\ cm^2$, $A_T = \left(4+4\sqrt{17}\right)\pi\ cm^2$,
   $V_T = \frac{32}{3}\pi\ cm^3$
9. $A_L = \frac{15\sqrt{3}}{4}\pi\ cm^2$, $A_T = \frac{5}{4}\left(5+3\sqrt{3}\right)\pi\ cm^2$
   $V_T = \frac{25\sqrt{3}}{6}\pi\ cm^3$
10. $A_L = 3\pi\ cm^2$, $A_T = 4\pi\ cm^2$, $V_T = \frac{2\sqrt{2}}{3}\pi\ cm^3$
11. $V_T = 12\ cm^3$
12. $V_T = 8\ cm^3$
13. $V_T = 12\sqrt{46}\ cm^3$
14. $V_T = \frac{560}{3}\ cm^3$
15. $A_B = 24\sqrt{3}\ cm^2$
16. $V_T = 24\pi\ cm^3$

17. $A_L = 70\pi\ cm^2$

18. $V_T = 12\pi\ cm^3$

19. $A_T = 48\pi\ cm^2$

20. $A_T = \frac{1}{2}\sqrt[3]{18\left(1+\sqrt{5}\right)^3 \pi V_T^{\,2}}$

EJERCICIO 36

1. $A = 64\pi\ cm^2,\ V = \frac{256}{3}\pi\ cm^3$
2. $V = 180\sqrt{5}\ \pi\ cm^3$
3. $V = 6\pi\ cm^3$
4. $V = 270\pi\ cm^3$
5. $A = 60\pi\ cm^2$
6. $A = 96\pi\ cm^2$
7. $V = \frac{28}{3}\pi\ cm^3$
8. $V = \frac{52}{3}\pi\ cm^3$
9. $V = 339\pi\ cm^3$,
   $A = 72\pi\ cm^2$
10. $V = \frac{211}{216}\pi\ cm^3$
11. $A = \frac{200}{3}\pi\ cm^2$
12. $n = 120º$
13. $V = 72\sqrt{3}\ \pi\ cm^3$
14. $r = \frac{9}{2}\ cm,\ A = 9\pi\ cm^2$

    $V = \frac{243}{2}\pi\ cm^3$

CAPÍTULO 11

EJERCICIO 37

Inciso 1)

a)

$sen\ A = \frac{2\sqrt{14}}{9}$	$cos\ A = \frac{5}{9}$	$tan\ A = \frac{2\sqrt{14}}{5}$
$ctg\ A = \frac{5\sqrt{14}}{28}$	$sec\ A = \frac{9}{5}$	$csc\ A = \frac{9\sqrt{14}}{28}$
$sen\ B = \frac{5}{9}$	$cos\ B = \frac{2\sqrt{14}}{9}$	$tan\ B = \frac{5\sqrt{14}}{28}$
$ctg\ B = \frac{2\sqrt{14}}{5}$	$sec\ B = \frac{9\sqrt{14}}{28}$	$csc\ B = \frac{9}{5}$

b)

$sen\ M = \frac{10\sqrt{149}}{149}$	$cos\ M = \frac{7\sqrt{149}}{149}$	$tan\ M = \frac{10}{7}$
$ctg\ M = \frac{7}{10}$	$sec\ M = \frac{\sqrt{149}}{7}$	$csc\ M = \frac{\sqrt{149}}{10}$
$sen\ N = \frac{7\sqrt{149}}{149}$	$cos\ N = \frac{10\sqrt{149}}{149}$	$tan\ N = \frac{7}{10}$
$ctg\ N = \frac{10}{7}$	$sec\ N = \frac{\sqrt{149}}{10}$	$csc\ N = \frac{\sqrt{149}}{7}$

c)

$sen\ A = \frac{2}{3}$	$cos\ A = \frac{\sqrt{5}}{3}$	$tan\ A = \frac{2\sqrt{5}}{5}$
$ctg\ A = \frac{\sqrt{5}}{2}$	$sec\ A = \frac{3\sqrt{5}}{5}$	$csc\ A = \frac{3}{2}$
$sen\ B = \frac{\sqrt{5}}{3}$	$cos\ B = \frac{2}{3}$	$tan\ B = \frac{\sqrt{5}}{2}$
$ctg\ B = \frac{2\sqrt{5}}{5}$	$sec\ B = \frac{3}{2}$	$csc\ B = \frac{3\sqrt{5}}{5}$

d)

$sen\ M = \frac{\sqrt{2}}{2}$	$cos\ M = \frac{\sqrt{2}}{2}$	$tan\ M = 1$
$ctg\ M = 1$	$sec\ M = \sqrt{2}$	$csc\ M = \sqrt{2}$
$sen\ N = \frac{\sqrt{2}}{2}$	$cos\ N = \frac{\sqrt{2}}{2}$	$tan\ N = 1$
$ctg\ N = 1$	$sec\ N = \sqrt{2}$	$csc\ N = \sqrt{2}$

Inciso 2)

a)

$\text{sen}\ \theta = \frac{2\sqrt{6}}{5}$	$cos\ \theta = \frac{1}{5}$	$tan\ \theta = 2\sqrt{6}$
$ctg\ \theta = \frac{\sqrt{6}}{12}$	$sec\ \theta = 5$	$csc\ \theta = \frac{5\sqrt{6}}{12}$
$sen\ \alpha = \frac{1}{5}$	$cos\ \alpha = \frac{2\sqrt{6}}{5}$	$tan\ \alpha = \frac{\sqrt{6}}{12}$
$ctg\ \alpha = 2\sqrt{6}$	$sec\ \alpha = \frac{5\sqrt{6}}{12}$	$csc\ \alpha = 5$

b)

$sen\ A = \frac{3\sqrt{13}}{13}$	$cos\ A = \frac{2\sqrt{13}}{13}$	$tan\ A = \frac{3}{2}$
$ctg\ A = \frac{2}{3}$	$sec\ A = \frac{\sqrt{13}}{2}$	$csc\ A = \frac{\sqrt{13}}{3}$
$sen\ B = \frac{2\sqrt{13}}{13}$	$cos\ B = \frac{3\sqrt{13}}{13}$	$tan\ B = \frac{2}{3}$
$ctg\ B = \frac{3}{2}$	$sec\ B = \frac{\sqrt{13}}{3}$	$csc\ B = \frac{\sqrt{13}}{2}$

c)

$sen\ N = \frac{1}{2}$ $cos\ N = \frac{\sqrt{3}}{2}$ $tan\ N = \frac{\sqrt{3}}{3}$

$ctg\ N = \sqrt{3}$ $sec\ N = \frac{2\sqrt{3}}{3}$ $csc\ N = 2$

$sen\ M = \frac{\sqrt{3}}{2}$ $cos\ M = \frac{1}{2}$ $tan\ M = \sqrt{3}$

$ctg\ M = \frac{\sqrt{3}}{3}$ $sec\ M = 2$ $csc\ M = \frac{2\sqrt{3}}{3}$

d)

$sen\ \theta = \frac{\sqrt{33}}{6}$ $cos\ \theta = \frac{\sqrt{3}}{6}$ $tan\ \theta = \sqrt{11}$

$ctg\ \theta = \frac{\sqrt{11}}{11}$ $sec\ \theta = 2\sqrt{3}$ $csc\ \theta = \frac{2\sqrt{33}}{11}$

$sen\ \alpha = \frac{\sqrt{3}}{6}$ $cos\ \alpha = \frac{\sqrt{33}}{6}$ $tan\ \alpha = \frac{\sqrt{11}}{11}$

$ctg\ \alpha = \sqrt{11}$ $sec\ \alpha = \frac{2\sqrt{33}}{11}$ $csc\ \alpha = 2\sqrt{3}$

e)

$sen\ \beta = \frac{\sqrt{6}}{4}$ $cos\ \beta = \frac{\sqrt{10}}{4}$ $tan\ \beta = \frac{\sqrt{15}}{5}$

$ctg\ \beta = \frac{\sqrt{15}}{3}$ $sec\ \beta = \frac{2\sqrt{10}}{5}$ $csc\ \beta = \frac{2\sqrt{6}}{3}$

$sen\ \alpha = \frac{\sqrt{10}}{4}$ $cos\ \alpha = \frac{\sqrt{6}}{4}$ $\tan\ \alpha = \frac{\sqrt{15}}{3}$

$ctg\ \alpha = \frac{\sqrt{15}}{5}$ $sec\ \alpha = \frac{2\sqrt{6}}{3}$ $csc\ \alpha = \frac{2\sqrt{10}}{5}$

f)

$sen\ A = \frac{4\sqrt{29}}{29}$ $cos\ A = \frac{\sqrt{377}}{29}$ $tan\ A = \frac{4\sqrt{13}}{13}$

$ctg\ A = \frac{\sqrt{13}}{4}$ $sec\ A = \frac{\sqrt{377}}{13}$ $csc\ A = \frac{\sqrt{29}}{4}$

$sen\ B = \frac{\sqrt{377}}{29}$ $cos\ B = \frac{4\sqrt{29}}{29}$ $tan\ B = \frac{\sqrt{13}}{4}$

$ctg\ B = \frac{4\sqrt{13}}{13}$ $sec\ B = \frac{\sqrt{29}}{4}$ $csc\ B = \frac{\sqrt{377}}{13}$

EJERCICIO 38

1.

$sen\ \alpha = -\frac{5}{13}$ $cos\ \alpha = \frac{12}{13}$ $tan\ \alpha = -\frac{5}{12}$

$ctg\ \alpha = -\frac{12}{5}$ $sec\ \alpha = \frac{13}{12}$ $csc\ \alpha = -\frac{13}{5}$

2.

$sen\ \alpha = -\frac{4\sqrt{65}}{65}$ $cos\ \alpha = -\frac{7\sqrt{65}}{65}$ $tan\ \alpha = \frac{4}{7}$

$ctg\ \alpha = \frac{7}{4}$ $sec\ \alpha = -\frac{\sqrt{65}}{7}$ $csc\ \alpha = -\frac{\sqrt{65}}{4}$

3.

$sen\ \beta = \frac{3\sqrt{13}}{13}$ $cos\ \beta = \frac{2\sqrt{13}}{13}$ $tan\ \beta = \frac{3}{2}$

$ctg\ \beta = \frac{2}{3}$ $sec\ \beta = \frac{\sqrt{13}}{2}$ $csc\ \beta = \frac{\sqrt{13}}{3}$

4.

$sen\ \omega = -\frac{\sqrt{2}}{2}$ $cos\ \omega = \frac{\sqrt{2}}{2}$ $tan\ \omega = -1$

$ctg\ \omega = -1$ $sec\ \omega = \sqrt{2}$ $csc\ \omega = -\sqrt{2}$

5.

$sen\ \alpha = -\frac{2}{3}$ $cos\ \alpha = -\frac{\sqrt{5}}{3}$ $tan\ \alpha = \frac{2\sqrt{5}}{5}$

$ctg\ \alpha = \frac{\sqrt{5}}{2}$ $sec\ \alpha = -\frac{3\sqrt{5}}{5}$ $csc\ \alpha = -\frac{3}{2}$

6.

$sen\ \alpha = -\frac{\sqrt{77}}{11}$ $cos\ \alpha = \frac{2\sqrt{11}}{11}$ $tan\ \alpha = -\frac{\sqrt{7}}{2}$

$ctg\ \alpha = -\frac{2\sqrt{7}}{7}$ $sec\ \alpha = \frac{\sqrt{11}}{2}$ $csc\ \alpha = -\frac{\sqrt{77}}{7}$

7.

$sen\ \beta = \frac{2\sqrt{22}}{13}$ $cos\ \beta = -\frac{9}{13}$ $tan\ \beta = -\frac{2\sqrt{22}}{9}$

$ctg\ \beta = -\frac{9\sqrt{22}}{44}$ $sec\ \beta = -\frac{13}{9}$ $csc\ \beta = \frac{13\sqrt{22}}{44}$

8.

$sen\ \omega = -\frac{\sqrt{65}}{65}$ $cos\ \omega = \frac{8\sqrt{65}}{65}$ $tan\ \omega = -\frac{1}{8}$

$ctg\ \omega = -8$ $sec\ \omega = \frac{\sqrt{65}}{8}$ $csc\ \omega = -\sqrt{65}$

9.

$sen\ \delta = \frac{5}{13}$ $cos\ \delta = -\frac{12}{13}$ $tan\ \delta = -\frac{5}{12}$

$ctg\ \delta = -\frac{12}{5}$ $sec\ \delta = -\frac{13}{12}$ $csc\ \delta = \frac{13}{5}$

10.

$sen\ \beta = -\frac{\sqrt{6}}{3}$ $cos\ \beta = -\frac{\sqrt{3}}{3}$ $tan\ \beta = \sqrt{2}$

$ctg\ \beta = \frac{\sqrt{2}}{2}$ $sec\ \beta = -\sqrt{3}$ $csc\ \beta = -\frac{\sqrt{6}}{2}$

11.

$sen\ \alpha = \frac{\sqrt{3}}{2}$ $cos\ \alpha = -\frac{1}{2}$ $tan\ \alpha = -\sqrt{3}$

$ctg\ \alpha = -\frac{\sqrt{3}}{3}$ $sec\ \alpha = -2$ $csc\ \alpha = \frac{2\sqrt{3}}{3}$

12.

$sen\ \alpha = -\frac{\sqrt{3}}{2}$ $cos\ \alpha = \frac{1}{2}$ $tan\ \alpha = -\sqrt{3}$

$ctg\ \alpha = -\frac{\sqrt{3}}{3}$ $sec\ \alpha = 2$ $csc\ \alpha = -\frac{2\sqrt{3}}{3}$

EJERCICIO 39

Inciso 1)

a) – *sen* 30° = –*cos* 60°

b) –*tan* 15° = –*ctg* 75°

c) *cos* 80° = *sen* 10°

d) *csc* 60° = *sec* 30°

e) *sec* 2° = *csc* 88°

f) –*sen* 60° 37’ 25” = –*cos* 29° 22’ 35”

g) –*ctg* 45° = –*tan* 45°

h) *tan* 74° 46’ 24” = *ctg* 15° 13’ 36”

i) –*cos* 84° 35’ = –*sen* 5° 25’

j) *sec* 39° 11’ 48” = *csc* 50° 48’ 12”

k) *csc* 53° = *sec* 37°

l) –*ctg* 48° = –*tan* 42°

m) *cos* 38° 54’ = *sen* 51° 6’

n) –*sen* 28° 35’ 24” = –*cos* 61° 24’ 36”

Inciso 2)

a) –*sen* 160°

b) –*ctg* 140°

c) *sec* 240°

d) *cos* 280°

e) –*tan* 345°

f) –*csc* 90°

g) *cos* 225° 15’ 46”

h) –*ctg* 176° 45’ 23”

i) *sec* 108° 32’

j) –*sen* 228°15’

Inciso 3)

a) –*sen* 20°

b) –*ctg* 20°

c) *cos* 80°

d) *tan* 45°

e) –*csc* 81° 27’ 48”

f) –*sec* 50°

g) –*sen* 55°

h) –*tan* 76° 34’ 42”

i) *cos* 68° 45’ 24”

j) *ctg* 20°

k) –*sec* 40°

l) –*csc* 31° 26’ 19”

Inciso 4)

a) 0.3090

b) 0.9657

c) 1.1034

d) 0.1219

e) 0.7536

f) 1.0187

g) 0.9261

h) 3.8208

i) 1.0170

j) 0.4975

Capítulo 12

Ejercicio 40

Grados	Radianes	sen	cos	tan	csc	sec	ctg
0°	0	0	1	0	No existe	1	No existe
30º	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	No existe	1	No existe	0
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
180°	π	0	-1	0	No existe	-1	No existe
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\frac{\sqrt{3}}{3}$
270°	$\frac{3\pi}{2}$	-1	0	No existe	-1	No existe	0
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\frac{\sqrt{3}}{3}$
315°	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
360°	2π	0	1	0	No existe	1	No existe

1. $\frac{\sqrt{3}}{2}$	5. $\frac{3}{16}$	9. 1	13. –1
2. $\frac{\sqrt{3}}{2}$	6. $\frac{1}{8}$	10. $\frac{\sqrt{3}}{4}$	14. 0
3. $\frac{3}{2}$	7. 9	11. 2	15. 2
4. 0	8. $\sqrt{2}$	12. 1	

16 a 20. No se incluye la solución por ser demostraciones.

CAPÍTULO 13

EJERCICIO 41

1. Amplitud: 2, Periodo: $\frac{2}{3}\pi$

   Desplazamiento de fase: $\frac{\pi}{6}$, $\frac{5}{6}\pi$

2. Amplitud: 2, Periodo: $\frac{1}{2}\pi$

   Desplazamiento de fase: 0, $\frac{1}{2}\pi$

3. Amplitud: $\frac{4}{3}$, Periodo: 3π

   Desplazamiento de fase: $-\frac{3}{4}\pi$, $\frac{9}{4}\pi$

4. Amplitud: 5, Periodo: 8π

   Desplazamiento de fase: -2π, 6π

5. Amplitud: 4

   Periodo: 2π

   Desplazamiento de fase: $\frac{3\pi}{4}$, $\frac{11}{4}\pi$

6. Amplitud: 3

   Periodo: π

   Desplazamiento de fase: 0, π

7. Amplitud: $\frac{3}{2}$

   Periodo: $\frac{2}{5}\pi$

   Desplazamiento de fase: $-\frac{3\pi}{10}$, $\frac{\pi}{10}$

8. Amplitud: $\frac{1}{3}$

   Periodo: 8π

   Desplazamiento de fase: $-\frac{4\pi}{3}$, $\frac{20\pi}{3}$

9. Amplitud: 1

   Periodo: 6π

   Desplazamiento de fase: 0, 6π

10. Periodo: $\frac{\pi}{2}$

    Asíntotas verticales: $\ldots, -\frac{\pi}{4}, \frac{\pi}{4}, \ldots$

    Desplazamiento de fase: no existe

11. Periodo: π

    Asíntotas verticales: $\ldots, -\frac{3}{4}\pi, \frac{\pi}{4}, \ldots$

    Desplazamiento de fase: $-\frac{\pi}{4}$ a la izq.

12. Periodo: $\frac{\pi}{3}$

    Asíntotas verticales: $\ldots, -\frac{\pi}{18}, \frac{5\pi}{18}, \ldots$

    Desplazamiento de fase: $\frac{\pi}{9}$ a la der.

13. Periodo: 2π

    Asíntotas verticales: $\ldots, \pi, 3\pi, \ldots$

    Desplazamiento de fase: 2π a la der.

14. Periodo: 4π

    Asíntotas verticales: $\ldots, 0, 4\pi, \ldots$

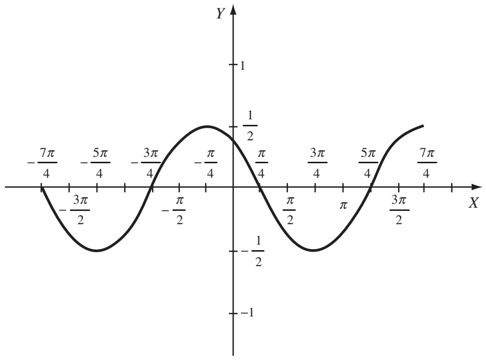
    Desplazamiento de fase: 2π a la der.

15. Periodo: π

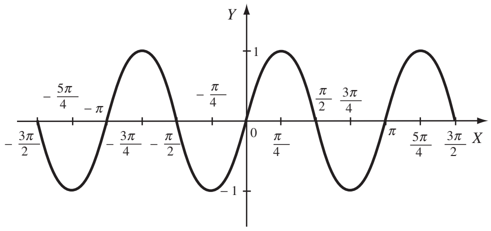
    Asíntotas verticales: $\ldots, \frac{1}{2}\pi, \frac{3}{2}\pi, \ldots$

    Desplazamiento de fase: π a la der.

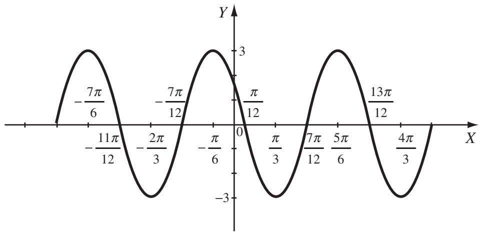
16.



17.



18.



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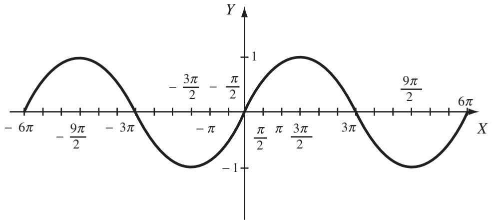

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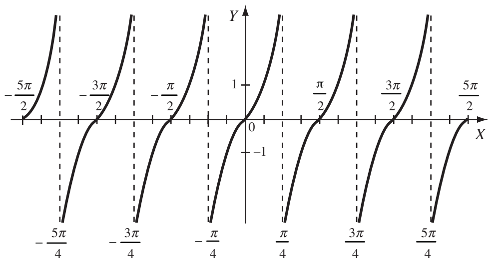

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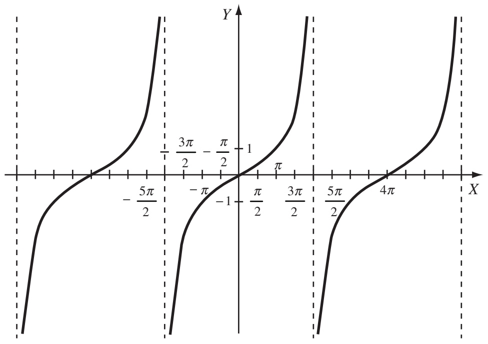
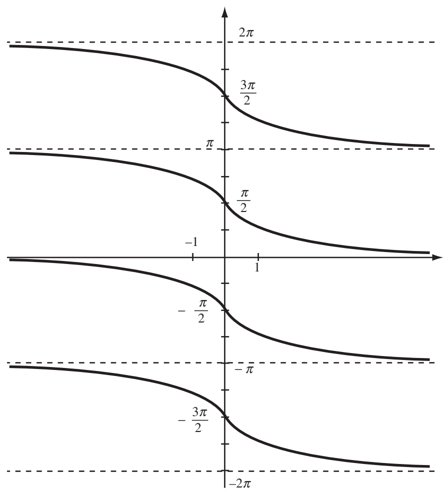
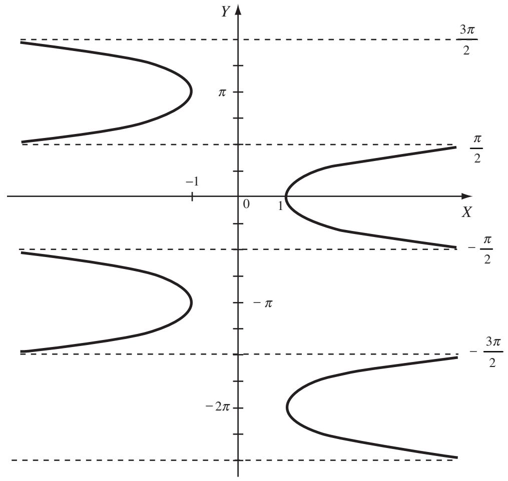


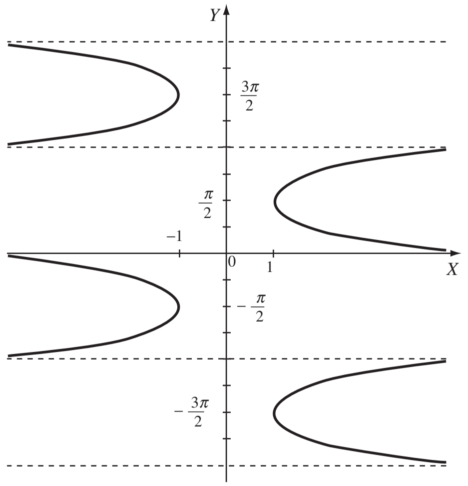
22.



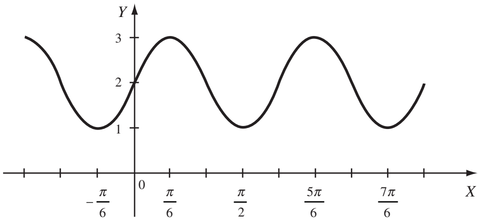
23.



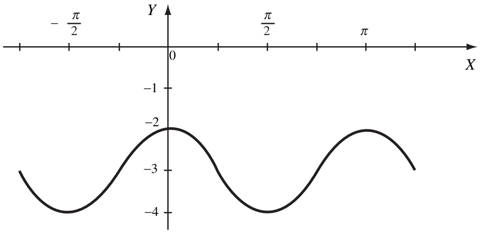
24.



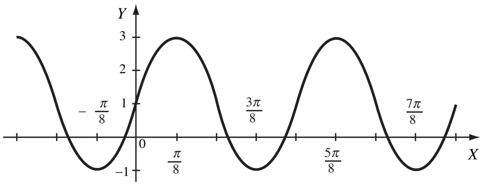
25.



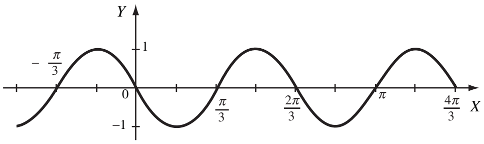
26.



27.



28.



CAPÍTULO 14

EJERCICIO 42

1 a 32. No se incluye la solución por ser demostraciones.

EJERCICIO 43

1. $\frac{\sqrt{3}}{2}$
2. $\frac{\sqrt{6}-\sqrt{2}}{4}$
3. $\frac{\sqrt{6}+\sqrt{2}}{4}$
4. -1
5. $-\sqrt{2}$
6. $-\frac{\sqrt{2}}{2}$
7. $-\sqrt{3}$
8. $-\sqrt{2}$
9. 1
10. 1
11. $\frac{1}{2}\left(\sqrt{3}sen\,\theta+cos\,\theta\right)$
12. $\frac{\sqrt{2}}{2}\left(sen\ x-cos\ x\right)$
13. $sen\ \beta$
14. $\frac{1}{tan\ x}=cot\ x$
15. $\frac{2}{\sqrt{3}cos\ \alpha-sen\ \alpha}$
16. $\frac{1-tan\ \beta}{1+tan\ \beta}$
17. $\frac{1}{2}\left(\sqrt{3}sen\ x-\cos x\right)$
18. $-\ sec\ 2\omega$
19. $-\ tan\ \alpha$
20. $\frac{\sqrt{2}}{2}\left(sen\,\theta+cos\,\theta\right)$

EJERCICIO 44

1. $-\left(2+\sqrt{3}\right)$
2. $2-\sqrt{3}$
3. $\sqrt{2}\left(1+\sqrt{3}\right)$
4. $-\sqrt{2}\left(1+\sqrt{3}\right)$
5. $2+\sqrt{3}$
6. $\frac{\sqrt{6}-\sqrt{2}}{4}$
7. $\sqrt{3}-2$
8. $-\left(\sqrt{6}-\sqrt{2}\right)$
9. $\sqrt{2}-\sqrt{6}$
10. $\frac{\sqrt{2-\sqrt{3}}}{2}$
11. $sen\ (\alpha+\beta)=\frac{\sqrt{13}}{65}$, $cos\ (\alpha+\beta)=-\frac{18\sqrt{13}}{65}$,

    $tan\ (\alpha+\beta)=-\frac{1}{18}$
12. $sen\ (\alpha-\beta)=-\frac{\sqrt{2}+\sqrt{6}}{4}$, $cos(\alpha-\beta)=\frac{\sqrt{6}-\sqrt{2}}{4}$,

    $tan\ (\alpha-\beta)=-\left(2+\sqrt{3}\right)$
13. Funciones del ángulo $(\alpha+\beta)$

    $sen(\alpha+\beta)=-\frac{\sqrt{3}}{9}\left(2+\sqrt{10}\right), cos(\alpha+\beta)=\frac{\sqrt{3}}{9}\left(\sqrt{5}-2\sqrt{2}\right)$

    $tan(\alpha+\beta)=3\sqrt{2}+2\sqrt{5}, ctg(\alpha+\beta)=\frac{2\sqrt{5}-3\sqrt{2}}{2}$,

    $sec(\alpha+\beta)=-\left(\sqrt{15}+2\sqrt{6}\right), csc(\alpha+\beta)=\sqrt{3}-\frac{1}{2}\sqrt{30}$

    Funciones del ángulo $(\alpha-\beta)$

    $sen\left(\alpha-\beta\right)=\frac{\sqrt{3}}{9}\left(2-\sqrt{10}\right), cos\left(\alpha-\beta\right)=-\frac{\sqrt{3}}{9}\left(\sqrt{5}+2\sqrt{2}\right)$

    $tan\left(\alpha-\beta\right)=2\sqrt{5}-3\sqrt{2}\ , ctg\left(\alpha-\beta\right)=\frac{2\sqrt{5}+3\sqrt{2}}{2}$

    $sec\left(\alpha-\beta\right)=\sqrt{15}-2\sqrt{6}\ , csc\left(\alpha-\beta\right)=-\sqrt{3}-\frac{1}{2}\sqrt{30}$

14 a 34. No se incluye la solución por ser demostraciones.

Ejercicio 45

1.

Funciones trigonométricas del ángulo $\frac{\pi}{8}$

$sen\ \frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$ $\quad ctg\ \frac{\pi}{8} = \sqrt{3+2\sqrt{2}}$

$cos\ \frac{\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad sec\ \frac{\pi}{8} = \sqrt{4-2\sqrt{2}}$

$tan\ \frac{\pi}{8} = \sqrt{3-2\sqrt{2}}$ $\quad csc\ \frac{\pi}{8} = \sqrt{4+2\sqrt{2}}$

Funciones trigonométricas del ángulo $\frac{3}{8}\pi$

$sen\ \frac{3}{8}\pi = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad ctg\ \frac{3}{8}\pi = \sqrt{3-2\sqrt{2}}$

$cos\ \frac{3}{8}\pi = \frac{\sqrt{2-\sqrt{2}}}{2}$ $\quad sec\ \frac{3}{8}\pi = \sqrt{4+2\sqrt{2}}$

$tan\ \frac{3}{8}\pi = \sqrt{3+2\sqrt{2}}$ $\quad csc\ \frac{3}{8}\pi = \sqrt{4-2\sqrt{2}}$

Funciones trigonométricas del ángulo $\frac{5}{8}\pi$

$sen\ \frac{5}{8}\pi = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad ctg\ \frac{5}{8}\pi = -\sqrt{3-2\sqrt{2}}$

$cos\ \frac{5}{8}\pi = -\frac{\sqrt{2-\sqrt{2}}}{2}$ $\quad sec\ \frac{5}{8}\pi = -\sqrt{4+2\sqrt{2}}$

$tan\ \frac{5}{8}\pi = -\sqrt{3+2\sqrt{2}}$ $\quad csc\ \frac{5}{8}\pi = \sqrt{4-2\sqrt{2}}$

Funciones trigonométricas del ángulo $\frac{7}{8}\pi$

$sen\ \frac{7}{8}\pi = \frac{\sqrt{2-\sqrt{2}}}{2}$ $\quad ctg\ \frac{7}{8}\pi = -\sqrt{3+2\sqrt{2}}$

$cos\ \frac{7}{8}\pi = -\frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad sec\ \frac{7}{8}\pi = -\sqrt{4-2\sqrt{2}}$

$tan\ \frac{7}{8}\pi = -\sqrt{3-2\sqrt{2}}$ $\quad csc\ \frac{7}{8}\pi = \sqrt{4+2\sqrt{2}}$

2.

Funciones trigonométricas del ángulo $\frac{\alpha}{2}$

$sen\ \frac{\alpha}{2} = \frac{\sqrt{8+2\sqrt{15}}}{4}$ $\quad ctg\ \frac{\alpha}{2} = \sqrt{31-8\sqrt{15}}$

$cos\ \frac{\alpha}{2} = \frac{\sqrt{8-2\sqrt{15}}}{4}$ $\quad sec\ \frac{\alpha}{2} = 2\sqrt{8+2\sqrt{15}}$

$tan\ \frac{\alpha}{2} = \sqrt{31+8\sqrt{15}}$ $\quad csc\ \frac{\alpha}{2} = 2\sqrt{8-2\sqrt{15}}$

Funciones trigonométricas del ángulo 2α

$sen\ 2\alpha = -\frac{\sqrt{15}}{8}$ $\quad ctg\ 2\alpha = -\frac{7\sqrt{15}}{15}$

$cos\ 2\alpha = \frac{7}{8}$ $\quad sec\ 2\alpha = \frac{8}{7}$

$tan\ 2\alpha = -\frac{\sqrt{15}}{7}$ $\quad csc\ 2\alpha = -\frac{8\sqrt{15}}{15}$

3.

Funciones trigonométricas del ángulo $\frac{\beta}{2}$

$sen\ \frac{\beta}{2} = \frac{3\sqrt{13}}{13}$ $\quad ctg\ \frac{\beta}{2} = -\frac{2}{3}$

$cos\ \frac{\beta}{2} = -\frac{2\sqrt{13}}{13}$ $\quad sec\ \frac{\beta}{2} = -\frac{\sqrt{13}}{2}$

$tan\ \frac{\beta}{2} = -\frac{3}{2}$ $\quad csc\ \frac{\beta}{2} = \frac{\sqrt{13}}{3}$

Funciones trigonométricas del ángulo 2β

$sen\ 2\beta = \frac{120}{169}$ $\quad ctg\ 2\beta = -\frac{119}{120}$

$cos\ 2\beta = -\frac{119}{169}$ $\quad sec\ 2\beta = -\frac{169}{119}$

$tan\ 2\beta = -\frac{120}{119}$ $\quad csc\ 2\beta = \frac{169}{120}$

4.

Funciones trigonométricas del ángulo $\frac{\omega}{2}$

$sen\ \frac{\omega}{2} = \frac{\sqrt{3}}{4}$ $\quad ctg\ \frac{\omega}{2} = -\frac{\sqrt{39}}{3}$

$cos\ \frac{\omega}{2} = -\frac{\sqrt{13}}{4}$ $\quad sec\ \frac{\omega}{2} = -\frac{4\sqrt{13}}{13}$

$tan\ \frac{\omega}{2} = -\frac{\sqrt{39}}{13}$ $\quad csc\ \frac{\omega}{2} = \frac{4\sqrt{3}}{3}$

Funciones trigonométricas del ángulo 2ω

$sen\ 2\omega = -\frac{5\sqrt{39}}{32}$ $\quad ctg\ 2\omega = \frac{7\sqrt{39}}{195}$

$cos\ 2\omega = -\frac{7}{32}$ $\quad sec\ 2\omega = -\frac{32}{7}$

$tan\ 2\omega = \frac{5\sqrt{39}}{7}$ $\quad csc\ 2\omega = -\frac{32\sqrt{39}}{195}$

5.

Funciones trigonométricas del ángulo $\frac{\alpha}{2}$

$sen\ \frac{\alpha}{2} = \frac{\sqrt{98+28\sqrt{7}}}{14}$ $\quad ctg\ \frac{\alpha}{2} = \frac{\sqrt{33-12\sqrt{7}}}{3}$

$cos\ \frac{\alpha}{2} = \frac{\sqrt{98-28\sqrt{7}}}{14}$ $\quad sec\ \frac{\alpha}{2} = \frac{\sqrt{42+12\sqrt{7}}}{3}$

$tan\ \frac{\alpha}{2} = \frac{\sqrt{33+12\sqrt{7}}}{3}$ $\quad csc\ \frac{\alpha}{2} = \frac{\sqrt{42-12\sqrt{7}}}{3}$

Funciones trigonométricas del ángulo 2α

$sen\ 2\alpha = -\frac{4\sqrt{3}}{7}$ $\quad ctg\ 2\alpha = -\frac{\sqrt{3}}{12}$

$cos\ 2\alpha = \frac{1}{7}$ $\quad sec\ 2\alpha = 7$

$tan\ 2\alpha = -4\sqrt{3}$ $\quad csc\ 2\alpha = -\frac{7\sqrt{3}}{12}$

6.

Funciones trigonométricas del ángulo α

$sen\ \alpha = \frac{2}{3}$ $\quad cos\ \alpha = -\frac{\sqrt{5}}{3}$ $\quad tan\ \alpha = -\frac{2\sqrt{5}}{5}$

7.

Funciones trigonométricas del ángulo $\frac{\beta}{2}$

$sen\ \frac{\beta}{2} = \frac{\sqrt{578+136\sqrt{17}}}{34}$ $\quad ctg\ \frac{\beta}{2} = -\sqrt{33-8\sqrt{17}}$

$cos\ \frac{\beta}{2} = -\frac{\sqrt{578-136\sqrt{17}}}{34}$ $\quad sec\ \frac{\beta}{2} = -\sqrt{34+8\sqrt{17}}$

$tan\ \frac{\beta}{2} = -\sqrt{33+8\sqrt{17}}$ $\quad csc\ \frac{\beta}{2} = \sqrt{34-8\sqrt{17}}$

Funciones trigonométricas del ángulo β

$sen\ \beta = -\frac{\sqrt{17}}{17}$ $\quad ctg\ \beta = 4$

$cos\ \beta = -\frac{4\sqrt{17}}{17}$ $\quad sec\ \beta = -\frac{\sqrt{17}}{4}$

$tan\ \beta = \frac{1}{4}$ $\quad csc\ \beta = -\sqrt{17}$

8.

Funciones trigonométricas del ángulo α

$sen\ \alpha = \frac{2\sqrt{5}}{5}$ $\quad tan\ \alpha = 2$ $\quad sec\ \alpha = \sqrt{5}$

$cos\ \alpha = \frac{\sqrt{5}}{5}$ $\quad ctg\ \alpha = \frac{1}{2}$ $\quad csc\ \alpha = \frac{\sqrt{5}}{2}$

9.

Funciones trigonométricas del ángulo $\frac{\beta}{2}$

$sen\ \frac{\beta}{2} = \frac{\sqrt{3}}{3}$ $\quad tan\ \frac{\beta}{2} = \frac{\sqrt{2}}{2}$ $\quad sec\ \frac{\beta}{2} = \frac{\sqrt{6}}{2}$

$cos\ \frac{\beta}{2} = \frac{\sqrt{6}}{3}$ $\quad ctg\ \frac{\beta}{2} = \sqrt{2}$ $\quad csc\ \frac{\beta}{2} = \sqrt{3}$

Funciones trigonométricas del ángulo β

$sen\ \beta = \frac{2\sqrt{2}}{3}$ $\quad tan\ \beta = 2\sqrt{2}$ $\quad sec\ \beta = 3$

$cos\ \beta = \frac{1}{3}$ $\quad ctg\ \beta = \frac{\sqrt{2}}{4}$ $\quad csc\ \beta = \frac{3\sqrt{2}}{4}$

10.

Funciones trigonométricas del ángulo ω

$sen\ \omega = -\frac{3}{5}$ $\quad ctg\ \omega = -\frac{4}{3}$

$cos\ \omega = \frac{4}{5}$ $\quad sec\ \omega = \frac{5}{4}$

$tan\ \omega = -\frac{3}{4}$ $\quad csc\ \omega = -\frac{5}{3}$

Funciones trigonométricas del ángulo 2ω

$sen\ 2\omega = -\frac{24}{25}$ $\quad ctg\ 2\omega = -\frac{7}{24}$

$cos\ 2\omega = \frac{7}{25}$ $\quad sec\ 2\omega = \frac{25}{7}$

$tan\ 2\omega = -\frac{24}{7}$ $\quad csc\ 2\omega = -\frac{25}{24}$

Funciones trigonométricas del ángulo 4ω

$sen\ 4\omega = -\frac{336}{625}$ $\quad ctg\ 4\omega = \frac{527}{336}$

$cos\ 4\omega = -\frac{527}{625}$ $\quad sec\ 4\omega = -\frac{625}{527}$

$tan\ 4\omega = \frac{336}{527}$ $\quad csc\ 4\omega = -\frac{625}{336}$

11 a 25. No se incluye la solución por ser demostraciones.

Ejercicio 46

1. $\frac{1}{2}\left[sen(2\alpha)+sen(2\beta)\right]$
2. $\frac{1}{2}\left[sen(105°)+sen(15°)\right]$
3. $-\frac{1}{2}\left[cos(2y)-cos(2\beta)\right]$
4. $\frac{1}{2}\left[cos\left(\frac{2}{3}\pi\right)+cos\left(\frac{1}{6}\pi\right)\right]$
5. $\frac{1}{2}\left[sen(120°)+sen(45°)\right]$
6. $-\frac{1}{2}\left[cos(45°)-cos(30°)\right]$
7. $\frac{1}{2}\left[sen(2x)-sen(2\alpha)\right]$
8. $\frac{1}{2}\left[cos(\pi)+cos\left(\frac{1}{6}\pi\right)\right]$
9. $\frac{1}{2}\left[sen(45°)-sen(30°)\right]$
10. $\frac{1}{2}\left[cos\left(\frac{11}{6}\pi\right)+cos\left(\frac{5}{3}\pi\right)\right]$
11. $-2\left[cos(4\alpha)-cos(2\alpha)\right]$
12. $\frac{5}{2}\left[sen(8\alpha)+sen(4\alpha)\right]$
13. $\frac{1}{2}\left[sen(90°)-sen(4°)\right]$
14. $\frac{1}{2}\left[cos\frac{1}{3}(2\alpha+5\beta)+cos\frac{1}{3}(2\alpha-5\beta)\right]$
15. $\frac{3}{2}\left[sen\left(\frac{19}{2}\alpha\right)+sen\left(\frac{17}{2}\alpha\right)\right]$
16. $\dfrac{2}{cos\frac{\pi}{2}+cos\frac{\pi}{6}}$
17. $\dfrac{sen3\alpha+sen\alpha}{sen3\alpha-sen\alpha}$
18. $\dfrac{2}{sen\pi-sen\frac{\pi}{2}}$
19. $\dfrac{\cos2\alpha-\cos2x}{\cos2\alpha+\cos2x}$
20. $\frac{1}{2}\left[sen(4\alpha)+sen(2\beta)\right]$

Ejercicio 47

1 a 14. No se incluye la solución por ser demostraciones.

Ejercicio 48

1. $2\left[sen(120°)\cdot cos(45°)\right]$
2. $2\left[cos\left(\frac{5}{2}\beta\right)cos\left(\frac{9}{2}\beta\right)\right]$
3. $2\left[sen(180°)cos(60°)\right]$
4. $-2\left[sen(4\theta)sen(\theta)\right]$
5. $2\left[cos(45°)cos(7°31')\right]$
6. $2\left[cos\left(\frac{1}{3}\pi\right)sen\left(\frac{1}{4}\pi\right)\right]$
7. $2\left[cos\left(\frac{1}{4}\pi\right)cos\left(\frac{1}{36}\pi\right)\right]$
8. $2\left[cos(30°)sen(5°)\right]$
9. $-2\left[\text{sen}\left(\frac{5}{12}\pi\right)\text{sen}\left(\frac{1}{3}\pi\right)\right]$
10. $2\left[cos(\beta)cos\left(\frac{1}{6}\pi\right)\right]$
11. $2\left[sen\left(\frac{7}{24}\pi\right)cos\left(\frac{1}{24}\pi\right)\right]$
12. $2\left[sen\left(\frac{3}{4}(\alpha+\beta)\right)cos\left(\frac{1}{4}(\alpha-\beta)\right)\right]$
13. $-2\left[sen(\alpha)sen\left(\frac{\pi}{4}\right)\right]$
14. $2\left[sen(\beta)cos\left(\frac{\pi}{8}\right)\right]$
15. $2\left[sen\left(\frac{3}{4}\pi\right)cos\left(\alpha-\frac{\pi}{8}\right)\right]$
16. $-2\left[sen\left(\frac{\alpha}{2}\right)sen\left(\frac{\beta}{2}\right)\right]$

EJERCICIO 49

1 a 12. No se incluye la solución por ser demostraciones.

EJERCICIO 50

1. $\frac{\pi}{4}, \frac{5}{4}\pi$
2. $\frac{\pi}{2}$, 36° 52’ 11”
3. $\frac{7}{12}\pi, \frac{23}{12}\pi$
4. $\frac{1}{4}\pi, \frac{5}{4}\pi$
5. $\frac{1}{6}\pi, \frac{5}{6}\pi$
6. $\frac{1}{3}\pi, \frac{5}{3}\pi$
7. $0, \pi, 2\pi$
8. $0, \pi, 2\pi, \frac{7}{6}\pi, \frac{11}{6}\pi$
9. $0, 2\pi$, 152° 44’, 207° 15’
10. $\frac{1}{4}\pi, \frac{7}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi$
11. $\frac{1}{2}\pi, \frac{1}{4}\pi, \frac{5}{4}\pi$
12. $\frac{7}{6}\pi, \frac{11}{6}\pi$
13. $\frac{1}{4}\pi, \frac{5}{4}\pi$
14. $\frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$
15. $\frac{1}{6}\pi, \frac{7}{6}\pi$
16. $\frac{1}{2}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi$
17. $\frac{1}{4}\pi, \frac{5}{4}\pi$
18. $0, \pi, 2\pi, \frac{2}{3}\pi, \frac{4}{3}\pi$
19. $\frac{1}{3}\pi, \frac{5}{3}\pi$
20. $0, \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi, \pi, 2\pi$
21. $0, \frac{2}{3}\pi, \frac{5}{3}\pi, \pi, 2\pi$
22. $\frac{1}{2}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi$
23. $\frac{3}{2}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$
24. $\frac{1}{3}\pi, \frac{5}{3}\pi$
25. $\frac{7}{6}\pi, \frac{11}{6}\pi, \frac{1}{3}\pi, \frac{5}{3}\pi$
26. $\frac{7}{6}\pi, \frac{1}{6}\pi$
27. $\frac{7}{12}\pi, \frac{11}{12}\pi$
28. $\frac{7}{6}\pi, \frac{11}{6}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi$
29. $\frac{1}{3}\pi, \frac{4}{3}\pi$
30. $\frac{1}{3}\pi, \pi, \frac{5}{3}\pi$

CAPÍTULO 15

EJERCICIO 51

1. $c = \sqrt{145}$, $A = 44° 54'$, $\angle C = 45° 6'$
2. $a = 2.11$, $c = 3.39$, $\angle C = 58°$
3. $c = 5.23$, $b = 7.24$, $\angle A = 43° 40'$
4. $b = 52.55$, $\angle A = 38° 11' 40''$, $\angle C = 51° 48' 20''$
5. $c = 13$, $b = 13\sqrt{2}$, $\angle C = 45°$
6. $a = 13.28$, $c = 18.28$, $\angle A = 36°$
7. $a = 12.51$, $\angle A = 33° 46' 46''$, $\angle C = 56° 13' 14''$
8. $a = 25.71$, $c = 22.9$, $\angle C = 41° 48'$
9. $a = 82.68$, $b = 100.36$, $\angle A = 55° 28'$
10. $c = 7.87$, $\angle A = 66° 39' 17''$, $\angle C = 23° 20' 43''$
11. $b = 22.36$, $c = 18.86$, $\angle C = 57° 33'$
12. $c = \sqrt{13}$, $\angle A = 29° 1' 1''$, $\angle C = 60° 58' 59''$
13. $a = 15.27$, $c = 17.19$, $\angle A = 41° 37'$
14. $b = 7.9$, $\angle A = 71° 33' 54''$, $\angle C = 18° 26' 5''$
15. $a = 6.28$, $b = 14.44$, $\angle C = 64° 11'$
16. $\angle A = 26° 33' 54''$, $\angle C = 63° 26' 6''$

17. $a = 5, b = 13, c = 12, \angle A = 22° 37' 11'', \angle C = 67° 22' 48''$
18. $a = 4, b = 5, c = 3, \angle A = 53° 7' 49'', \angle C = 36° 52' 11''$
19. $\angle A = \angle C = 45°$
20. $\angle A = 19° 28' 16'', \angle C = 70° 31' 44''$

Ejercicio 52

1. 288.4 m
2. 4.2 m
3. 38° 44′ 4″, 1.65m
4. $\left(10\sqrt{2}+1\right)\ m$
5. 54° 8′
6. 52.07 m
7. 11.25 m
8. a) 53.6 m , b) 59.1 m, c) 22.6 m
9. 53° 7′, 3 m
10. 21° 47′, 14 dm
11. $L = \frac{\pi R}{90}\left[180 - \cos^{-1}\left(\frac{R-r}{l}\right)\right] + \frac{\pi r}{90}\cos^{-1}\left(\frac{R-r}{l}\right) + 2\sqrt{l^2 - \left(R-r\right)^2}$
12. sí

Capítulo 16

Ejercicio 53

1. $a = 20.9, c = 14.7, \angle A = 79° 1'$
2. $b = 52.4, a = 47.7, \angle B = 79° 16'$
3. $b = 21.03, a = 46.9, \angle C = 67° 44'$
4. $b = 86.21, c = 66.87, \angle B = 76° 39'$
5. $a = 23.35, c = 25.23, \angle A = 67°$
6. $b = 17.09, c = 22.3, \angle C = 99°$
7. $c = 9.43, \angle B = 57° 58' 51'', \angle C = 90° 1' 8''$
8. $a = 19.8, \angle A = 118° 23' 35'', \angle B = 26° 21' 24''$
9. $c = 15.11, \angle A = 40° 5' 50'', \angle C = 83° 19' 9''$
10. $b = 11.4, \angle A = 46° 14' 25'', \angle B = 66° 24' 34''$
11. $\angle A = 31° 48' 52'', \angle B = 34° 12' 58'', \angle C = 113° 58' 10''$
12. $\angle A = 27° 25' 16'', \angle B = 44° 1' 54'', \angle C = 108° 32' 50''$
13. $\angle A = 52° 17' 24'', \angle B = 44° 33' 55'', \angle C = 83° 8' 41''$
14. $\angle A = 48° 20' 58'', \angle B = 36° 42' 37'', \angle C = 94° 56' 23''$
15. $c = 15.3, \angle A = 46° 39' 8'', \angle B = 65° 20' 52''$
16. $b = 37.07, \angle A = 47° 7' 45'', \angle C = 56° 52' 15''$
17. $a = 46.05, \angle B = 34° 5' 24'', \angle C = 110° 54' 36''$
18. $c = 15.65, \angle A = 41° 52' 18'', \angle B = 82° 7' 42''$

Ejercicio 54

1. $\overline{AB} = 369.95\ m$
2. 1.76 cm
3. 30.34 km
4. 19.4 km
5. 8.03 m
6. 4.7 cm
7. 322.92 km
8. 307.4 m
9. 29.07 km
10. 180.37 m
11. 29.7 cm

12 a 13. No se incluye la solución por ser demostraciones.

Capítulo 17

Ejercicio 55

1. $z = \sqrt{17} cis 345° 37' 49''$
2. $z = 2cis\ 30°$
3. $z = 2\sqrt{2}\ cis\ 135°$
4. $z = 5cis\ 0°$
5. $z = 3cis\ 270°$
6. $z = \frac{5}{6}\ cis\ 53° 7' 48''$
7. $z = cis\ 315°$
8. $z = cis\ 150°$
9. $z_1 \cdot z_2 = \sqrt{26}\ cis\ 75°$
10. $z_2 \cdot z_4 = \sqrt{26}\ cis\ 165°$
11. $z_1 \cdot z_3 = 2\sqrt{2}\ cis\ 105°$
12. $z_1 \cdot z_2 \cdot z_3 = 2\sqrt{26}\ cis\ 135°$
13. $z_1 \cdot z_3 \cdot z_4 = 4cis\ 240°$
14. $\frac{z_1}{z_4} = cis\ 270°$
15. $\frac{z_2}{z_4} = \frac{\sqrt{26}}{2}\ cis\ 255°$
16. $\frac{z_1}{z_3} = \frac{\sqrt{2}}{2}\ cis\ 345°$
17. $\frac{z_1 \cdot z_2}{z_3} = \frac{\sqrt{26}}{2}\ cis\ 15°$
18. $\frac{z_2}{z_1 \cdot z_4} = \frac{\sqrt{13}}{2}\ cis\ 210°$

19. $\dfrac{z_2 \cdot z_3}{z_1 \cdot z_4} = \sqrt{13}\ cis\ 270º$

20. $\dfrac{z_1 \cdot z_2 \cdot z_3}{z_4} = 2\sqrt{13}\ cis\ 0^c$

21. $z^2 = 9cis\ 240º$

22. $z^4 = 81cis\ 100º$

23. $z^3 = 125cis\ 45º$

24. $z_1 = 4cis\ 30º$, $z_2 = 4cis\ 210º$

25. $z_1 = 2cis\ 20º$, $z_2 = 2cis\ 80º$, $z_3 = 2cis\ 140º$, $z_4 = 2cis\ 200º$
$z_5 = 2cis\ 260º$, $z_6 = 2cis\ 320º$

26. $z_1 = cis\ 60º$, $z_2 = cis\ 180º$, $z_3 = cis\ 300º$

27. $(z \cdot z_1)^2 = 36cis\ 120º$

28. $z_1 = 2cis\ 30º$, $z_2 = 2cis\ 150º$, $z_3 = 2cis\ 270º$

29. $28cis\ 100º$

30. $z_1 = 4cis\ 10º$, $z_2 = 4cis\ 130º$, $z_3 = 4cis\ 250º$